Secondary School Teaching of Nonlinear Phenomena

PhD Theses

Meszéna Tamás

Supervisor: Dr. Tél Tamás
professor emeritus

ELTE TTK PhD School for Physics
Head of the School: Dr. Gubicza Jenő

Physics Education PhD Program
Head of the Program: Dr. Nguyen Quang Chinh

Eötvös Loránd University
Faculty of Science

2021

„I accept chaos, I'm not sure whether it accepts me."
Bob Dylan

Introduction

Contemplating the history of pedagogy, one may observe that the 'curriculum' seemed to be so clear and generally accepted over long periods of time that the question of what and how students should be taught has been completely ignored. Then these periods were typically followed by intensive transformation phases when previous practices were significantly changed. Undoubtedly, we live in the latter today.

The reason for changes is the unexpected development of science and its transformative effect on our whole lives. The curriculum change as a result of the new NAT is also related to this fact. Nevertheless, this process poses a serious challenge to developers of learning contents. One would need solid knowledge, a broad vision, and the courage to be able to make a change.

One cannot teach what science has already surpassed, but one cannot omit new, important results. However, it is not possible to continuously increase the amount of material either, and it is very difficult to shorten or leave out any part of it. We bitterly squeeze a little space for the new knowledge which would usually require a whole course of prior knowledge.

These are the countless problems that physics educators face on a daily basis, and we have only considered the scientific side of the subject. We have not mentioned, and we will not talk here, about the social, psychological, lifestyle aspects.

Several new advances in physics are sure to play a bigger role in our future. I would like to join this process with my research, which examines a few teachable phenomena of nonlinear physics.

Theses

1. Investigation of the bouncing of a point-like ball on a staircase using a realistic coefficient of restitution [1],[2]

It was once mentioned in an Austrian high school textbook, along with several other well-known chaotic phenomena, that the bouncing dynamics of a ball on a staircase is chaotic. [A] I have shown that a school study to this issue can be used to reinvestigate several important notions (e.g., energy loss) and to understand novel concepts (e.g., attractor, quasi-periodic motion).

At first glance, the problem we examine seem to be extremely simple: the motion of a point-like ball bouncing down on an infinitely long, right angled staircase, without taking air resistance into consideration. A one (or maybe two) demonstration class was developed on the motion mentioned.

During bouncing, the ball loses mechanical energy during the collision, which is considered as the coefficient of restitution *k*. Getting more and more lower on the stairs, gravitational energy replaces the energy loss. In my experience, thinking through the energy relationships of a ball bouncing off stairs with an imperfectly elastic collision helps students for understanding energy conservation and transformation at a higher level.

The elements of motion can be given, in a way that students can follow, according to the law of oblique bending and the rules of inelastic collision. Surprisingly, however, the complete motion cannot be described by a simple formula because the equations are nonlinear. These are relatively easy to solve numerically with the help of a computer leading to interesting results.

The vast majority of the parameter values are quasi-periodic, repeating themselves with small deviations. The motion is insensitive to initial conditions and 'forgets' its initial state due to collision losses. This gives students an illustrative example of what an attractor is, especially in our case, a quasi-periodic attractor.

It is surprising that the simpler, periodic motions can only be realized for well-defined cases with easily calculable collision coefficient values. Moreover, the representation of the collision coefficient values for pure periodic motions, which bounce once on each step, or on every second, third, etc., can remind us of the energy levels of the hydrogen atom. A novel, mind-forming discovery for students is the discrete spectrum appearing in mechanics.

To study the movement, we have created an online simulation program that not only visualize the first few bounces, but also calculates the characteristic values of an arbitrary number of bounces, such as the bounce location and the bounce rate, and the number of steps jumped. The program also plots the phase spaces that can be generated from these features. Students can modify the parameters while using the program. The popularity of its use among students can be shown by the fact that the website was accessed more than 2200 times in the last four years.

No evidence of chaos was found when examining the movement features, however the complexity of the trajectories and the strong presence of quasi-periodic movement gives students an understanding that a movement can be complex even if it is not chaotic.

2. Bouncing of a point-like ball on a staircase at high energy loss (low coefficient of restitution) [1],[2]

Examination of this domain helps students to understand additional concepts typical of nonlinear phenomena (e.g., higher order cycles, coexisting attractors, domains of attraction).

If the value of the coefficient of restitution falls below the value typical for a single bounce on each step, then the ball will bounce twice on the steps during its movement. If the coefficient of restitution decreases further, then we may find a purely periodic motion where bouncing over one step and bouncing on the same step occur alternately. Such a motion is called a second-order cycle.

Reducing this value further, we get to movements where the ball bounces twice, three times, etc. on a step. By learning about this movement, students understand that higher order cycles can also be attractors.

For small coefficients of restitution, new phenomena can be observed in the motion of the bouncing ball. The vertical velocity component may decrease so fast that even after an infinite number of bounces the ball does not reach the edge of the step (the motion is stalled), while the horizontal velocity component of the ball remains constant throughout, i.e., it reaches the end of the step in a finite time. This is basically called sliding.

I managed to make the students understand that sliding can also be considered a kind of attractor, because if the ball bounces infinitely many times on one step, i.e., a sliding has occurred, then the rest of the movement cannot continue across further steps. We can say that the ball has arrived at the sliding attractor.

There exists an interval of coefficient of restitution within which the bouncing and the pure bouncing on the infinite stairs appear alternately, which is interesting because it would seem logical that if for a given coefficient of restitution the bouncing on the stairs ceases, then for smaller coefficients the same situation would be found even more so.

From this, students can understand that two types of attractors may characterize a movement. In this case, the quasi-periodic attractor of bouncing is accompanied by the attractor of long-lasting sliding, and the two exist together.

This is possible because in these cases, the location of the first bounce on the steps and the value of the vertical initial velocity determine which of the two types of attractors will characterize the movement. The domains of attraction of the two attractors show a banded structure in the 'initial velocity - first bounce location' plane.

By following this process, I provide a new opportunity for students to understand the concept of the domain of attraction (where the boundary is non-fractal in nature due to the absence of chaos).

Finally, there is a certain coefficient of collision below which long-term bouncing movements disappear completely and a sliding motion develops under any initial condition. From this, students learn that strong frictional loss leads to overdamping of the motion.

3. The necessity of teaching the concept of quasi-periodic motion and its experiences in secondary schools [I], [II]

The reason for the choice of this topic is that in the secondary school curriculum, in addition to progressive and periodic motion, students also hear about disordered motion (Brownian motion), but quasi-periodic motion is not mentioned as an intermediate possibility. I have shown that the knowledge of this motion (which is much simpler than chaos, but still very complex), usefully complements the teaching of mechanics and provides an opportunity to understand phenomena not yet taught (such as the conical pendulum, the rotation of Mercury's orbit, or the Foucault's pendulum).

Between November 2019 and February 2020, I taught a short one-hour lesson (including tests) on the concept of quasi-periodic motion to 52 students in three groups.

Through the known motions (perpendicular vibrations, Foucault's pendulum), I will show during the education how they are similar to periodic motions and why they are still different. Then, I will explain the concept of quasi-periodic motion by examining in detail the motion of a bouncing ball on a staircase. Teaching this topic, as defined above, fills important gaps in secondary education.

I used the introduction to the concept of quasi-periodic motion to apply the PER (Physics Education Research) method to improve and test the effectiveness of education. I used analogies in my teaching (perfectly elastic bouncing, various collisions), looked for illustrative examples (Foucault's pendulum, Mercury's perihelion rotation) and employed students both in real life, manually (bouncing a ball on stairs) and through computer simulations (Geogebra animation, ‘bouncing on the stairs’ program).

I measured students' prior knowledge and understanding of what they had heard by means of tests taken at the beginning and end of the lesson. I analyzed the results of the tests and used them to modify and improve the test in several stages. In the final evaluation, only the results of the teaching programs carried out using the current form of the test are included.

The results of the tests show that the curriculum does indeed help students to understand the essence of quasi-periodic motion. On average, the results more than doubled, from 20.4% to 42.8%.

4. Presentation of the concept and phenomena of chaos at the secondary school level in a single teaching lesson [3], [4]

The teaching of chaos phenomena is not part of the secondary school curriculum. Examining the textbooks of several European countries, I found only a few mentions. Publications in Hungarian are mainly concerned with the teaching of chaos in specialized classes or groups. [B] - [M] I have shown that it is possible to understand the essence of chaos in public education within a single lesson.

I thought about two basic questions: why and how to teach this subject in public education.

The why question can be answered briefly and convincingly:

- because it is important, one of the defining physics results of the last decades, and one that has affected a surprisingly large number of fields,

- because it is interesting and serves well to synthesise the teaching of physics. I found that students in grades 11-12 responded well to the content, even if it was more complex than in previous topics.

The question of how is much more complex.

First, the previously learned movement types have to be organized in terms of regularity and periodicity. It must be made clear that the movements taught at school are simple basic cases that can only be achieved under a number of restrictive conditions. In reality, however, these conditions are never perfectly fulfilled, but at most the deviation is negligible for a give description.

I have prepared a teaching material that presents the basic features of chaotic movement in a single lesson.

I discuss the basic concepts of chaos with the help of illustrative examples. Is the movement of a pendulum clock, the rotation or orbit of the Earth periodic? To what extent can these movements be predicted? I will show that we encounter many phenomena in our environment that have in common that they are not periodic (not even quasi-periodic), unpredictable and have complex geometries. A new approach for students is to ask themselves to think about movements previously thought to be simply periodic, are they really periodic? At what precision can they be considered periodic?

It turned out a great experience to make students discover that there are several steps (quasi-periodic, chaotic) between perfectly regular and completely irregular movements. This realization is like hearing for the first time that the division of finite and infinite quantities can be further refined, since infinity has several distinct degrees.

I have found that mechanistic determinism is deeply rooted in our thinking. It is not easy to accept that very small initial inaccuracies can sometimes, in the not-too-distant future, lead to complete uncertainty.

We have a similarly strong belief, and this is unnoticed by students, that simple systems can be described by simple equations, complex systems by complex equations. What is surprising about chaos is that we discover the complex behavior of systems that can be described by simple equations.

I have shown that, with the teaching material I have used, these misconceptions can be resolved and pointed in the right direction.

It will significantly broaden students' understanding of why it is necessary to represent motion in phase space rather than in terms of position-time and velocity-time functions for these motions.

With the considerations described, I first attempt to delimit the content (non-periodic, unpredictable, fractal geometry motion, phase space, attractor) that should be considered as part of the basic curriculum when teaching chaotic phenomena in secondary schools.

5. Comparison and simulation of simple periodic and chaotic motions [3], [4]

In this tutorial for students who are particularly interested in chaotic motion, I will emphasize that the purely periodic motion of a mathematical pendulum can be easily made chaotic with a variety of small changes, e.g. the oscillating pendulum, the double pendulum, the pendulum swinging on an elastic fiber, etc. I will show that it is useful to compare the periodic motion characteristics learned in the core material with the quantities that characterize the modified pendulum motion, and that computer simulation can be used for this purpose.

I prove that it is worth considering the motion of the pendulum excited by time-dependent torque as a new example of chaotic motions. I will then introduce students to the concepts used to describe chaotic motion, and to the method and program that allow the motion to be studied.

For example, the maintenance of pendulum motion that is damped by losses can be solved by mechanical excitation that is very easily thought of as time-dependent. Sinusoidal time dependence can be regarded as a simple base case, which is not unfamiliar to high school students, who have encountered it in harmonic oscillatory motion. Excitation in this case is considered in terms of the torque acting on the suspension.

I use computer programs for detailed examinations. We will frequently confront with the well-known problem of missing mathematical knowledge (differential calculus, differential equations). I illustrate this with illustrative examples and analogies. We will consider how, in the pre-GPS and pre-radio era, the route of a ship could be plotted on a map from regularly recorded values of speed magnitude and heading, if the starting point was known.

In my experience, the Dynamics Solver program, which is free to download and excellent for numerically solving differential equations [C], is a good tool for mapping the characteristics of motion. It is possible to use the program without a detailed discussion of the theory of differential equations. Its application and operation can be understood by simulating the motions discussed earlier in the course.

By using a computer simulation, I illustrate to students how movement occurs under different conditions. As a starting point, I tried to choose parameter values close to reality. The students were interested when I changed the parameter values.

A major difficulty of the study is that we do not know how much changing the parameter values will significantly modify the nature of the resulting motion. They learn that a sense of reality and diligent work lead to results.

The description of the motion of a pendulum excited by time-dependent torque has not been published in this form. Examining the motion of many similar (oscillating, double, magnetic) modified pendulums is very useful in teaching because they are all chaotic, yet they differ in detail, making them an excellent practice opportunity.

Together with the students, we were able to find typical chaotic attractors with a nice, well-defined shape over a relatively wide range of parameters. The attractor is strongly dependent on the friction parameter, and its shape changes significantly as the friction parameter changes.

We have shown that by varying the parameters, non-chaotic domains can also be found, and students will realize how much more complicated chaotic motion is than periodic one.

6. Self-similarity implies a relation between holograms and chaos [5]

A not the best known but essential property of a hologram is that the whole image can be reconstructed from a single part of it, i.e. essentially all the information is contained in a small part of the hologram. An image reconstructed from a small detail may be poorer in light and more out of focus, but the whole shape can always be reconstructed. In my experience, this property offers an interesting parallel with the self-similarity of fractal structure typically seen in the description of chaotic phenomena.

I have also compiled a single lesson on this topic, which can be incorporated into the curriculum after learning about the hologram phenomenon.

A part of a mathematical fractal (e.g., a Koch curve) is exactly, geometrically similar to a smaller part, or the whole shape. Fractals, or fractal-like structures, obtained in the description of chaotic phenomena do not usually contain exact mathematical similarity, but essentially identical structures may be observed in them over increasingly smaller ranges of dimensions, i.e. they have self-similarity.

I am looking at two slightly different meanings of the concept of self-similarity in the two cases, and the relationship analyzed here is thought-provoking. The self-similarity of fractals can also be interpreted in such a way that the whole can be reconstructed from a small detail.

Meanwhile, chaotic movement is rather characterized by the fact that from the small details, novel, fundamental structures of the whole movement are revealed. The fractal structure appears less frequently in real space (as in the case of the magnetic pendulum) and is usually encountered in phase space.

So, in addition to self-similarity, the fractal structures of chaotic motion contain other information, because of their not entirely exact similarity, which only shows itself at a larger scale.

On the other hand, even if the self-similarity of the Koch curve is perfect, we can still discover a surplus in the whole curve compared to its parts, since the fractal dimension characterizes the spatial filling of the whole curve.

Students are interested to see the relationship between the part and the whole time after time. When discussing the issue, we review or discuss the characteristics of the hologram and the general properties of fractals, the concept of fractal dimension, as necessary.

The relationship between chaos and the hologram, which arises as a consequence of the phenomenon of self-similarity, has not yet been explored. Examining the similarities and commonalities observed in these different fields not only leads to a better understanding of both phenomena, but also helps students to develop an open-minded thinking that is worthwhile in approaching the subject of our study.

Summary

I am convinced that the teaching of nonlinear and chaotic phenomena at the secondary school level will become natural in the near future.

I would like to facilitate this process by using the internationally recognized method of didactics (Physics Education Research) to explore how the basic concepts of this new subject can be taught effectively and efficiently.

When textbook authors start to collect the content through which they want to introduce students to the theory of chaos, it will be useful to have any experience that can provide an objective measure of the acceptability of these concepts.

I have been commissioned by the Gelsey Vilmos Pedagogical Institute of the Diocese of Szeged-Csanád to write a new physics textbook together with a team. My task was to lead the working group and write the main text for half of the lessons. As part of this, I wrote one lesson on chaos. This would be the first national publication of chaos theory in a high school physics textbook. The manuscripts for the 9-10-11 textbooks and workbooks were completed, but the project is currently on hold.

I will continue to explore the teachability of the characteristics of nonlinear processes in the coming years, , as part of my five-year master pedagogue program launched in January 2020.

Own publications related to the theses

1. Gruiz M, Meszéna T, Tél T: Kaotikus vagy csak összetett? - Labdák pattogása lépcsőn, *Fizikai Szemle 2016/4.*, pp. 128.-136.
2. M. Gruiz, T. Meszéna, T. Tél: Chaotic or just complicated? Ball bouncing down the stairs, *European Journal of Physics 38, 055003(15), (2017)*
3. Meszéna T: Fraktálok és káosz, *A fizika, matematika és művészet találkozása az oktatásban, kutatásban, Konferenciakötet*, ELTE Budapest, 2013, pp. 153.-158.
4. T. Meszéna: Chaos at High School ICPE-EPEC 2013 Prague, Proceedings pp. 533-540
5. Meszéna Tamás: Rész és egész: a részek viszonya az egészhez a káoszjelenségekben és a hologramban, *Természettudomány tanítása korszerűen és vonzóan, Konferenciakötet*, ELTE Budapest, 2011, pp. 306.-310.

Oral Presentations

I. T. Meszéna: „A PER Experiment to Support an Important Concept in Modern Physics: The Quasi-periodic Motion", GIREP-ICPE-EPEC-MPTL 2019 CONFERENCE Celebration of Eötvös Year 2019 "Teaching-learning contemporary physics, from research to practice", Budapest 2019.

II. Meszéna T: Olasz fizikatanár továbbképzés Udinében, Scuola Nazionale per Insegnanti sulla Fisica Moderna SNI-FM2017, Universita degli Studi di Udine 2017. szeptember 4-9. „Pendolo caotico: un problema di fisica moderna"

Bibliography not directly related to the theses

A) Nussbaumer A and Nussbaumer P 2017 *Basiswissen, Physik-Compact 5* (Vienna: Österreichischer Bundesverlag Schulbuch GmbH) p 12

B) Csernovszky Zoltán: Az iránytű harmonikus rezgésétől kaotikus mozgásáig, *Fizikai Szemle 2017/6.*, pp. 198.-204.

C) Z. Csernovszky, P. Nagy, P. Tasnádi: Investigation of chaos in the absence of programming skill, *Canadian Journal of Physics, Special Issue 2020, Volume 98, Number 6, (593-605)* Journal link

D) Nagy Péter, Tasnádi Péter: Fraktálok világa - játékos tudomány, *Gradus (a Neumann János Egyetem online folyóirata) Vol. 6, No. 3 (2019)*

E) Nagy Péter, Tasnádi Péter: Fizikai modellalkotás – gondolatok egy versenyfeladat kapcsán, Matematikát, fizikát és informatikát oktatók (MAFIOK) 43. országos konferenciája Konferenciakötet 2019, pp. 87-98.

F) Nagy Péter, Tasnádi Péter: Rugósinga dinamikai vizsgálata - egy fizika versenyfeladat kaotikus utóélete, *Dunakavics (A Dunaújvárosi Egyetem online folyóirata) 2019. VII. évfolyam VIII. szám* pp. 31-46

G) Nagy Péter, Tasnádi Péter: Dynamics Solver - egy hatékony eszköz a káosz kutatásában és tanításában, *Matematikát, fizikát és informatikát oktatók (MAFIOK) 41. országos konferenciája, Konferenciakötet, szerk: Talata I., Szent István Egyetem, Ybl Miklós Építéstudományi Kar, Budapest, 2017.*, pp. 169-178.
H) Nagy Péter, Tasnádi Péter: Tálban guruló golyó kaotikus viselkedése, *Gradus 3, No. 2 (2016)*, pp. 120-136.
I) Tóthné Juhász Tünde, Gócz Éva: Káosz egy tálban (2014); (PDF dokumentum + Dynamics Solver szimulációk, RAR fájba tömörítve, 0,6 Mb),
J) Szatmári-Bajkó, Ildikó (2010) Káosz, rend, látvány. A káosztudomány ismertetésének lehetősége IKT-eszközökkel a középiskolai oktatás keretében. Iskolakultúra, 20 (1). pp. 116-131. ISSN 1215-5233
K) I.Szatmáry-Bajkó: Handicraft and aesthetic experience in teaching chaos physics TEACHING PHYSICS INNOVATIVELY 2015 konferenciakötete p.15
L) Stonawski Tamás: A digitális technika sodrában: káosz a tálban, Természettudomány tanítása korszerűen és vonzóan, Konferenciakötet, ELTE Budapest, 2011, pp. 311.-314.
M) Jaloveczki József: Numerikus módszerek a diákköri munkában, Fizikatanítás tartalmasan és érdekesen, Konferenciakötet, ELTE Budapest, 2010, pp. 303.-310.
N) Meszéna Tamás – Mező Tamás: Fizika 11. (manuscript in preparation)