From Pressure Maps and Wind Velocity to Northern Lights and Other Fascinating Phenomena on the Rotating Earth

Owing to the presence of the Coriolis effect, the rotation of the Earth has a multitude of surprising consequences that make the mechanics of the atmosphere or the oceans different from that of a fluid in a container. Since the Coriolis effect also captures the imagination of screenwriters, contributing to the continual exposure of students to bogus science, it is important to address these ideas in the physics classroom. This paper assumes that students are familiar with the Coriolis force acting on moving bodies in a rotating reference frame, and suggests a possible algebra based approach. The information encoded in pressure maps is used to obtain wind velocity, and the result is applied in various contexts to learn about tilting sea surfaces, northern lights and fluids flowing around phantom obstacles.

The Earth's rotation in films

Since there are numerous websites listing instances of pseudoscience in movies, a couple of representative examples will suffice to illustrate the necessity of classroom treatment.

The newscast announcing an enormous hurricane-like storm over Canada in *The Day After Tomorrow* shows a cloud system spiraling clockwise. In my experience, students with a background in geography can tell that − could a hurricane ever form over Canada at all − on the northern hemisphere it should certainly rotate the other way. Students reliably remember what they have learnt about low-pressure systems, that is why they so readily give credit to the drain hole fallacy. This fallacy appears in *Escape Plan*, too, where the inmates in a maximum secure prison are not even told where on the globe they are. The hero observes the water swirling in the toilet bowl and concludes that the prison is situated somewhere on the northern hemisphere. In the episode *Bart versus Australia* of *The Simpsons*, the behaviour of drains occasions diplomatic embroilment. It all starts with Bart Simpson refusing to accept that drains rotate differently on the two hemispheres. His intelligent sister Lisa points out that he should simply try and see, that is, she formulates the principle that experiment is the way to verify a scientific hypothesis. However, in spite of some writers of The Simpsons having degrees in science, they do not actually try for themselves, and we are shown water consistently draining differently. In the episode *Die Hand Die Verletzt* of *X-Files* even wrong science is remembered wrongly and, as evidence that occult powers are at work, agent Moulder claims that the counterclockwise rotation of the water in a drinking fountain would otherwise be characteristic of the southern hemisphere.

Wind direction and wind speed

While the Coriolis effect itself is often introduced[1,2], demonstrated and interpreted at high school level, too, its applications to atmospheric motions generally take a calculus based approach[3,4,5]. The use of pressure maps, however, offers a convenient elementary treatment.

High school students are aware that air currents are determined by the distribution of atmospheric pressure, and physics students have presumably seen isobar charts in the geography class. Although an isobar chart is the most widely known graphical representation of pressure distribution, it is not easy to interpret the information conveyed by the curves. Before applying pressure maps to learn about wind speed and direction, it is essential to familiarize students with pressure maps. There are a lot of educational websites[6] that offer student activities to introduce the pressure gradient force, that is, the force acting on an air parcel owing to pressure difference. In order to proceed, we will need the following result:

If the pressure difference over a small distance Δx is Δp, then the force acting on the air contained in a volume V between two surfaces A normal to Δx is

$$F_p = -A\Delta p = -A\Delta x \cdot \frac{\Delta p}{\Delta x} = -V \cdot \frac{\Delta p}{\Delta x}. \qquad (1)$$

The second factor is the pressure gradient that can be read from an isobar chart.

When students are shown an isobar chart and asked what the wind direction is, they usually conclude that it is perpendicular to the curves. However, some charts have the wind direction, too, revealing that the wind blows almost parallel to the isobars, as illustrated in Figure 1.

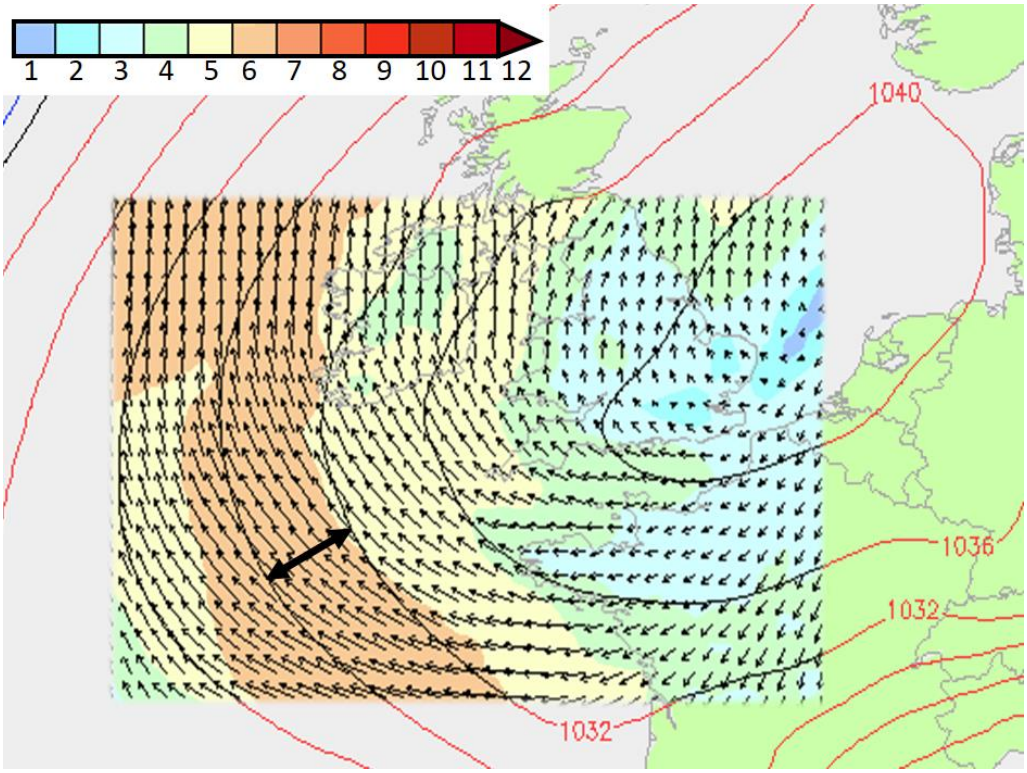

Fig. 1. An isobar chart of northwestern Europe, combined with a map of surface (ground level) wind directions and speeds. The isobars are labelled in hPa, and wind speeds are represented on the Beaufort scale[7]. (Credit: weatheronline.co.uk)

The intuitive answer, therefore, is in error, and there are two reasons for that. First, the conclusion that the wind blows from the high-pressure region towards the low-pressure region disregards the Coriolis effect. The other error is more subtle and yet more profound: Even though the distribution of atmospheric pressure is responsible for winds, pressure distribution only changes very slowly in time, and large-scale wind patterns of fairly constant speed and direction persist for long periods. Since the moving air does not accelerate (with respect to the Earth's surface), wind should not be interpreted as a result of one force or another: it is determined by an equilibrium of forces[8].
For simplicity, consider the wind blowing at a high altitude above the ground, where frictional effects can be disregarded, as shown in the left panel of Figure 2.

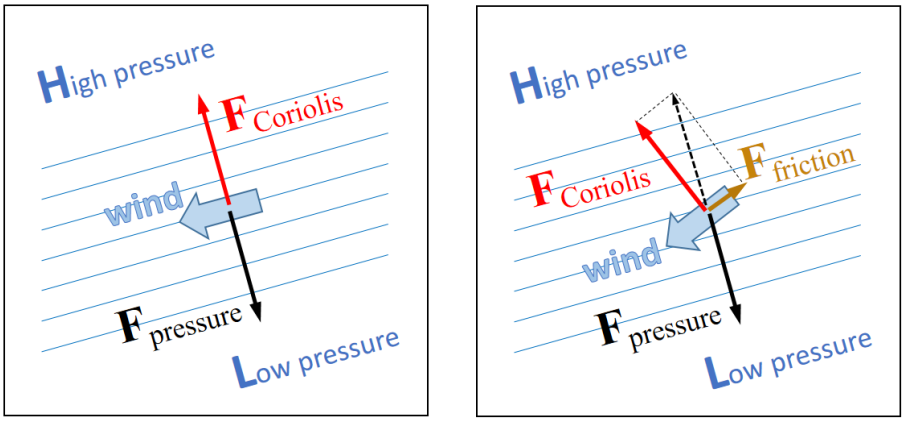

Fig. 2. The equilibrium of the forces acting on an air parcel in the absence of friction at high altitudes (left), and close to the surface where frction is present (right). The parallel lines represent the isobars.

The pressure gradient force (1) is thus balanced by the Coriolis force, expressed as

$$F_C = mv \cdot 2\Omega \sin \varphi$$

where Ω stands for the magnitude of the angular velocity of the Earth's rotation and φ is geographic latitude. In the case of equilibrium, this is equal and opposite to the pressure gradient force (1):

$$V \cdot \frac{\Delta p}{\Delta x} = mv \cdot 2\Omega \sin \varphi \, .$$

Hence, by replacing m/V with the density ρ, the wind speed

$$v = \frac{1}{\rho \cdot 2\Omega \sin\varphi} \cdot \frac{\Delta p}{\Delta x} \tag{2}$$

is obtained. Since the Coriolis force acts sideways on a moving object, the pressure gradient force is also sideways, which implies that wind blows along the isobars (called geostrophic wind, in meteorological terms) rather than across them.

Note that this conclusion only applies at high altitudes. The wind velocity vectors representing surface wind in Figure 1 make an angle with the isobars: at low elevations friction cannot be disregarded, and the equilibrium of forces is as shown in the right panel of Figure 2. Since the pressure gradient force is now greater in magnitude than the Coriolis force, equation (2) will only provide a rough upper estimate for the wind speed. In Figure 1, the thick black arrow at about 47° latitude marks a distance of approximately 200 km between two isobars 4 hPa apart. Substitution in (2), using an air density of 1.3 kg/m^3 delivers a speed of

$$v < \frac{1}{1.3 \cdot 2 \cdot 7.3 \cdot 10^{-5} \cdot \sin 47°} \cdot \frac{400}{200000} \frac{\mathrm{m}}{\mathrm{s}} = 14 \frac{\mathrm{m}}{\mathrm{s}}.$$

This would classify as a 7 on the Beaufort scale[7]. In reality, Figure 1 only indicates a level of 6.

The topography of pressure surfaces

Although the counterintuitive result of wind blowing along the isobars rather than across them is remarkable enough, by investigating pressure distribution in a little more depth we can also explain some further unexpected behaviour of the atmosphere and the seas.

A closer look at pressure maps will reveal that isobars are only used in ground level pressure charts. The maps representing upper level pressure distributions are different: they show lines of constant altitude for some specific isobaric surface. Meteorologist call these curves isohypses[9]. Figure 3 is a 500-hPa isohypse chart, that is, it can be interpreted as a chart showing the topographic contour lines of the surface enclosing the lower half of the atmospheric air.

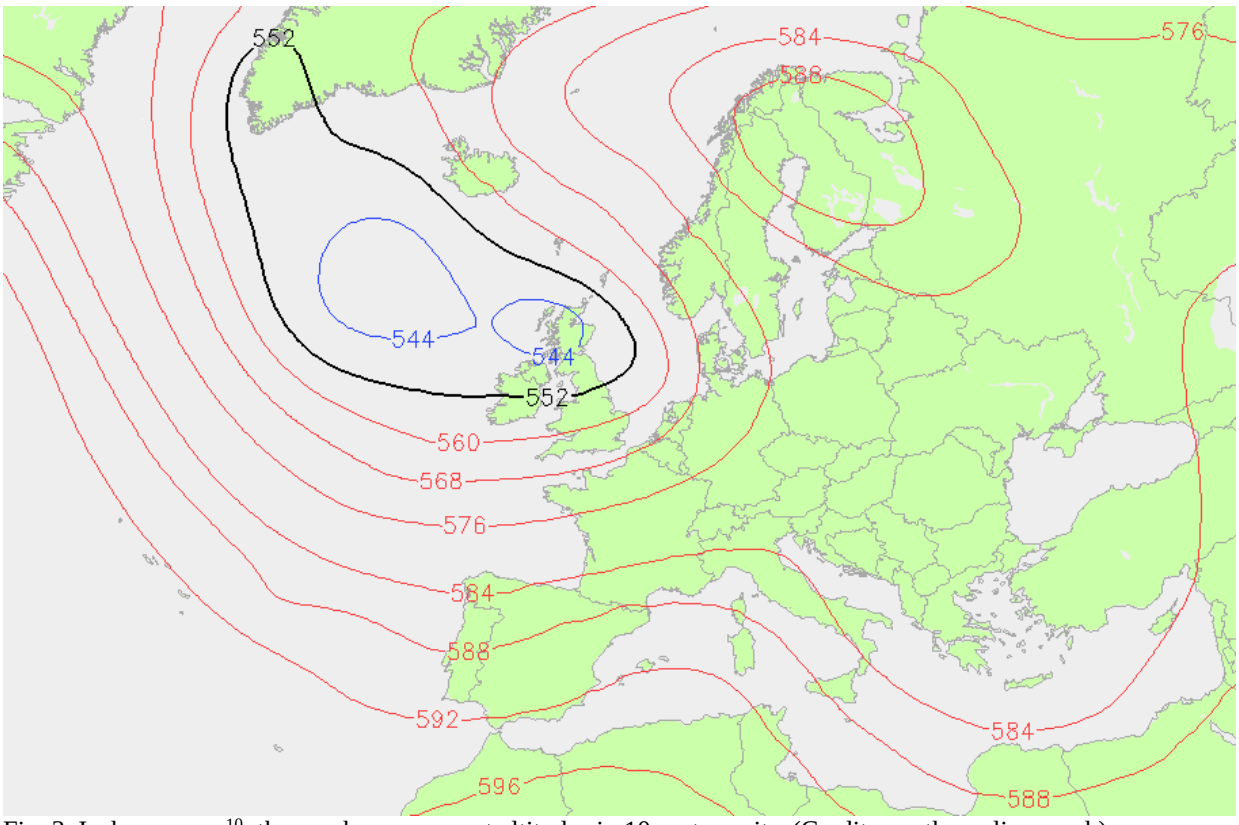

Fig. 3. Isohypse map[10]: the numbers represent altitudes in 10-metre units. (Credit: weatheronline.co.uk)

In order to see why these curves are more helpful than isobars, consider the acceleration owing to the pressure gradient force (1), transformed a little further:

$$\frac{F_p}{m} = -\frac{V}{m} \cdot \frac{\Delta p}{\Delta x} = -\frac{1}{\rho} \cdot \frac{\Delta p}{\Delta x} = -\frac{1}{\rho} \cdot \frac{\Delta p}{\Delta z} \cdot \frac{\Delta z}{\Delta x},$$

where z stands for altitude. Students know from basic fluid mechanics that the pressure at a depth h in a lake is $p = p_0 + \rho g h$ where p_0 is the pressure at the surface. To generalise, we can observe that pressure decreases with altitude at a rate proportional to the density of the fluid. Density may vary with position, but in this way density will cancel out of the equation, and there remain only the known value of g and the slope of the isobaric surface:

$$\frac{F_p}{m} = -\frac{1}{\rho} \cdot \frac{\Delta p}{\Delta z} \cdot \frac{\Delta z}{\Delta x} = -\frac{1}{\rho} \cdot (-\rho g) \cdot \frac{\Delta z}{\Delta x} = g \cdot \frac{\Delta z}{\Delta x}.$$

Under equilibrium conditions, this acceleration, expressed in terms of the slope of the isobaric surface, can again be set equal to the Coriolis acceleration, to yield a new expression for the speed:

$$v = \frac{1}{2\Omega \sin\varphi} \cdot g \cdot \frac{\Delta z}{\Delta x}. \tag{3}$$

For example, consider the region of the English Channel in Figure 3. Δz can be read from the contour lines and Δx can be measured on a geographic map. Figure 4 shows the region magnified, also indicating wind direction and the necessary geographic information.

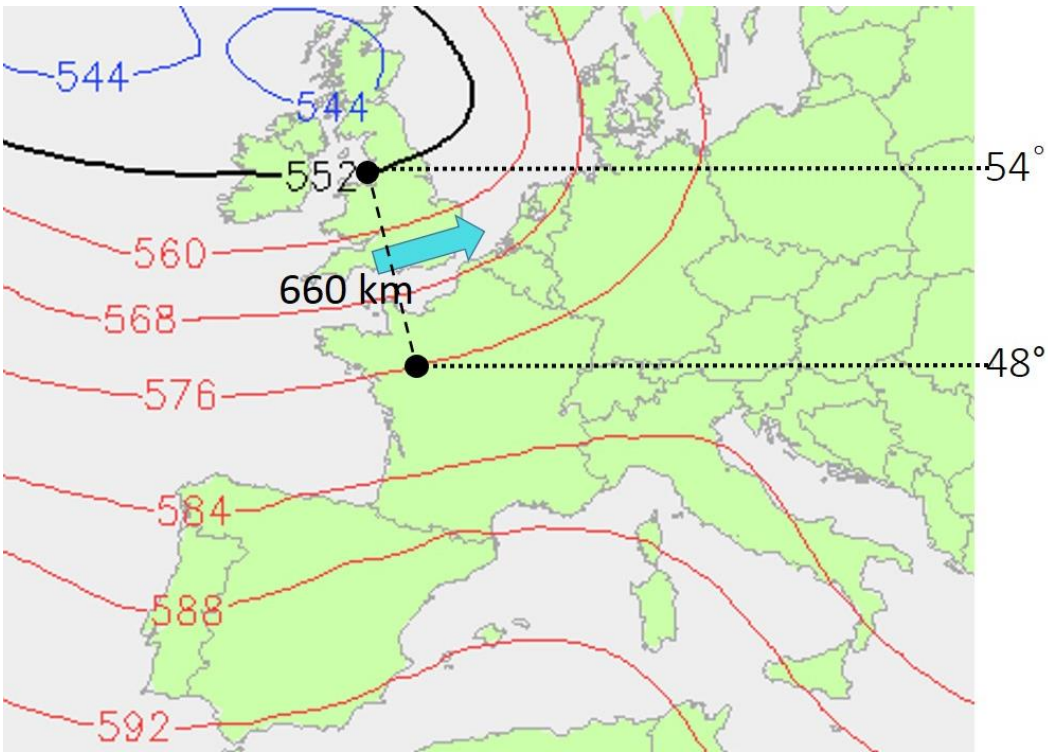

Fig. 4. Isohypse map with geographic data and wind direction.

With these data, the wind speed at a height of about 6 km above the English Channel is

$$v = \frac{1}{2\Omega \sin\varphi} \cdot g \cdot \frac{\Delta z}{\Delta x} = \frac{1}{2 \cdot 7.3 \cdot 10^{-5} \sin 51°} \cdot 9.8 \cdot \frac{240}{6.6 \cdot 10^5} \frac{\mathrm{m}}{\mathrm{s}} \approx 30 \frac{\mathrm{m}}{\mathrm{s}}.$$

Figure 5 shows a streamline map of actual wind velocities at 500 hPa height over the same region, a few hours earlier. Observe that at this high altitude the direction of the wind is, indeed, along the curves.

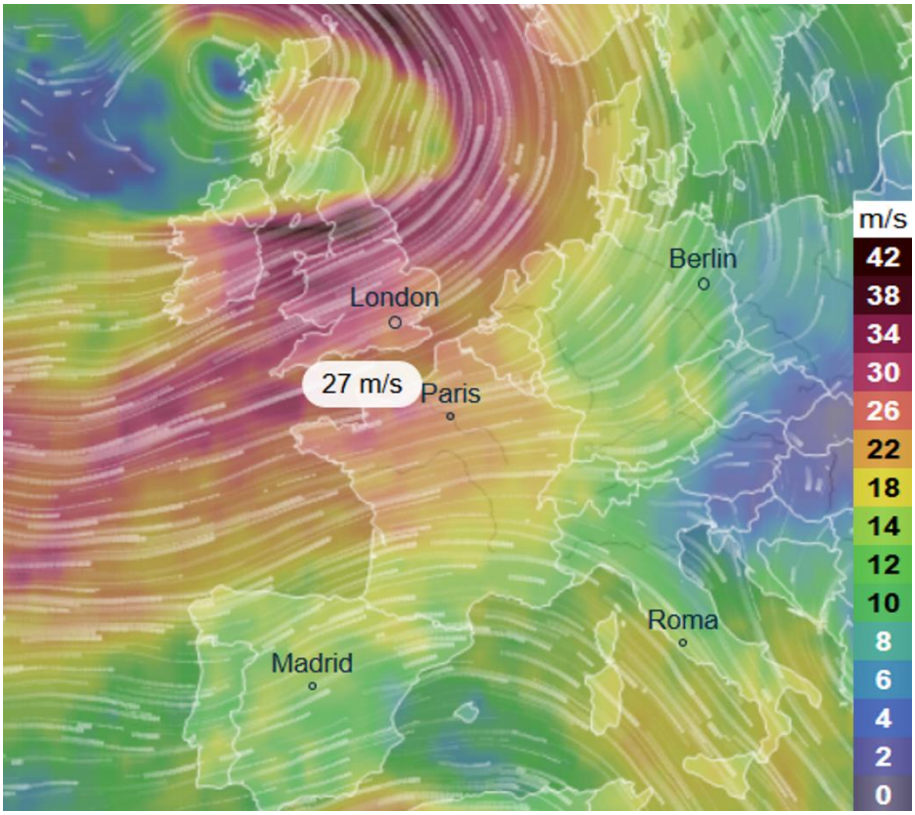

Fig. 5. Streamline map of 500 hPa wind velocity. (Credit: www.ventusky.com)

Where sea level is not level

Note that velocity depends (on geographic latitude and) on the slope of the pressure surfaces. This is a conclusion that lends itself to further discussion. One interesting consequence is obtained if the result (3) is applied to the oceans. Evidently, in the case of the sea there exists a natural isobaric surface: that of the sea level where the pressure equals the atmospheric pressure. The expression (3) thus implies that a current in dynamic equilibrium involves a tilted surface. This is indeed the case with the Gulf Stream, a current that everyone is familiar with since it even features in the popular media, in the context of climate change scenarios.

For example, consider the section of the Gulf Stream where it flows towards the northeast at 36° northern latitude. The width of the stream is in the order of 100 m, and it flows at a rate of about 1 m/s. These data provide an order-of-magnitude estimate for the tilt of the sea surface across the Gulf Stream. The numerical value of the slope expressed from (3) is

$$\frac{\Delta z}{\Delta x} = \frac{2\Omega \sin\varphi \cdot v}{g} = \frac{2 \cdot 7.3 \cdot 10^{-5} \cdot 0.59 \cdot 1}{10} = 8.6 \times 10^{-6} \approx 10^{-5}.$$

That means a rise of about 1 m over a distance of 100 km northwest to southeast: in good agreement with values obtained by satellite altimetry, as represented in Figure 6.

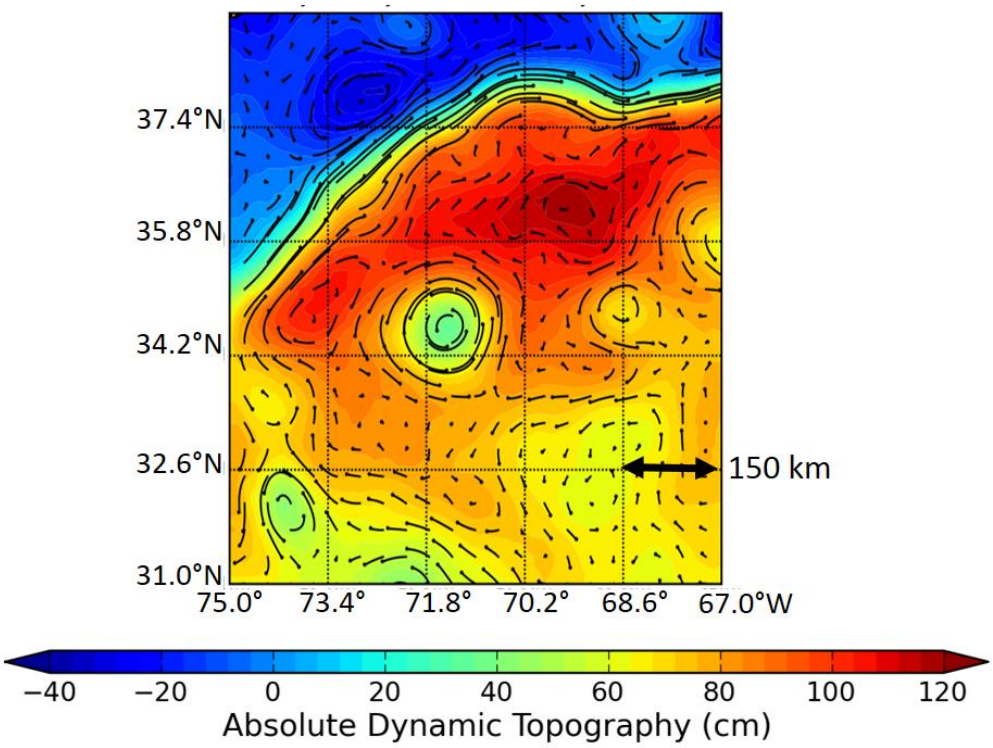

Fig. 6. Map of sea surface topography obtained from satellite measurements. The Gulf Stream is the narrow band separating the blue and light red regions. (Credit: Gerald Dibarboure)

Fluid curtains and phantom columns

Another interesting consequence follows when density can be considered uniform in a fluid (be that air or water). In that case pressure decreases with height at the same rate, $-\rho g$, along a vertical line erected at any point of the surface. Thus pressure surfaces have the same slope all along the vertical line, which results in the same horizontal velocity all along the vertical line[11].

It should be borne in mind that this conclusion only applies to fluids of uniform density that rotate fast enough for the Coriolis force to be important. The invariance of rotated fluids under vertical translation can also be demonstrated in the laboratory. The picture of Figure 7 shows a water tank placed on a rotating platform in the von Kármán Laboratory for Environmental Flows at ELTE University, Budapest. In the rotating water, a drop of dye has been injected. The same dye injected in a stationary tank would form a cloud showing turbulence in all three dimensions. Rotation, however, makes a great difference: the dye spreads along vertical surfaces resembling curtains.

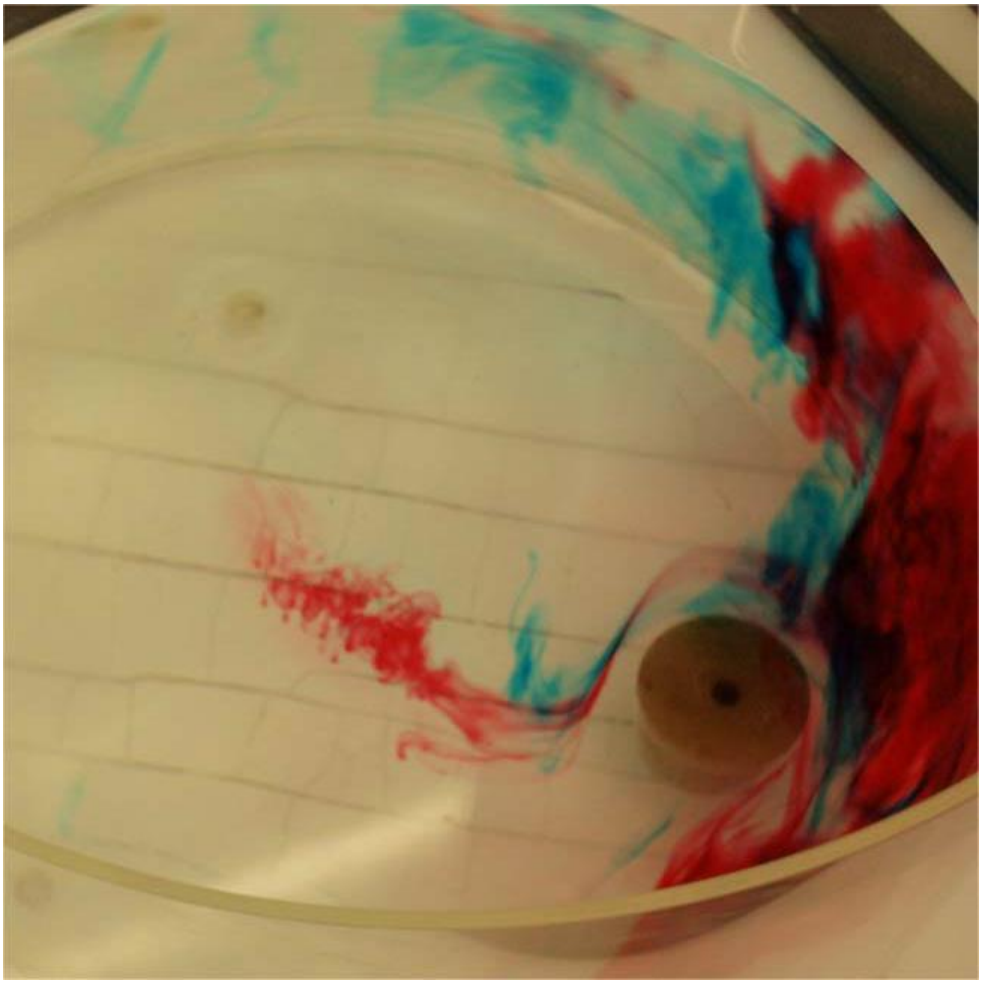

Fig. 7. Dye curtains in a rotating water tank. Demonstration in the von Kármán Laboratory for Environmental Flows.

Figure 7 demonstrates more than that: the most surprising behaviour of a rotating fluid is observed if an obstacle is placed at the bottom of the tank. Although the obstacle is much lower than the water level in the tank, the dye will not flow over the obstacle. The dye curtains not only flow around it at the bottom where the true obstacle is but also flow around an imaginary column over the obstacle.

Fluid curtains and the column remaining at rest over an obstacle (a so called Taylor column) can both be observed in nature, too: Polar lights are emitted by ionised gases in the upper atmosphere. If density is uniform, the rate of pressure decrease with height, $\Delta z/\Delta x$, is the same at all altitudes, therefore the velocity of gas particles situated along a vertical line is independent of the altitude. They will all move identically, that is, along identical trajectories, which results in a display of curtain-like shapes[12], as illustrated in Figure 8.

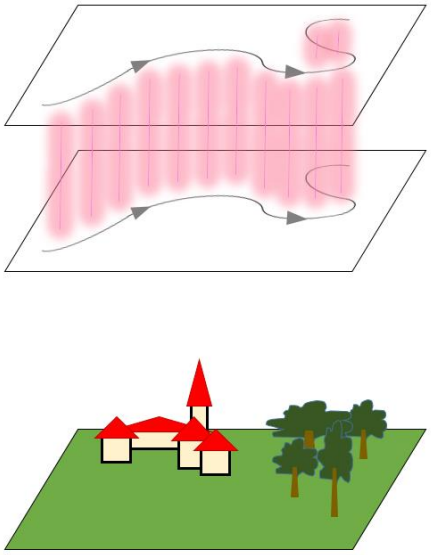

Fig. 8. When density is uniform, curtain-like shapes of polar lights result from the invariance of the flow under a vertical translation.

For a natural Taylor column, the most frequently cited example is the photograph in Figure 9, showing the behaviour of clouds over the Pacific island of Guadalupe. The clouds are a lot higher than the island, but the well known pattern of the von Kármán vortex street reveals that the wind flows around the stationary air column over the island, as if it had hit an obstacle.

Fig. 9. Kármán vortex street over the island of Guadalupe[13]. (Credit: NASA/GSFC/JPL, MISR Team)

The example of the summer ice retreat on the Chukchi Sea is not so widely known, but no less instructive. The Chukchi Sea lies to the north of the Bering Strait, bounded by the coasts of Alaska and Siberia (and by the rim of the continental shelf on the north). The Herald Shoal rises about 20 metres above the seabed that has a roughly uniform depth of 50 metres. (Figure 10).

Fig. 10. The Chukchi Sea[14] with the Herald Shoal drawn in.

In summer, a warm current arriving from the Bering Strait regularly melts the ice cover of the Chukchi Sea. Research[15,16] revealed that the ice retreat follows the same pattern year by year. Actually, even ship logs of 19th century whalers contain the observation that warm water advances like a tongue on each side of the Herald Shoal, and when the ice has melted at the sides and even beyond the shoal, there is still an ice cover over the shoal itself, for a long time. Temperature measurements show a well defined front at the edge of the shoal, and so do measurements of salinity. Under the persisting ice cover over the shoal, there is colder and less saline old water, while on either side there is the new water brought by the warm and salty current. Measurements also reveal that the density of water in the Chukchi Sea is quite uniform. All parameters meet the conditions required for the formation of a Taylor column.

1. J. Higbie, ”Simplified approach to Coriolis effects”, *The Physics Teacher* 18, 459−460 (September 1980)

2. Alpha E. Wilson, ”Jogging on a Carousel”, *The Physics Teacher* 49, 570−571 (February 2011)

3. Ned Mayo, ”A Hurricane for Physics Students”, *The Physics Teacher* 32, 148−154 (March 1994)

4. Amit Tandon, John Marshall, ”Einstein’s Tea Leaves and Pressure Systems in the Atmosphere”, *The Physics Teacher* 48, 96−99 (February 2010)

5. Richard M. Heavers, Rachel M. Dapp, ”The Ekman Layer and Why Tea Leaves Go to the Center of the Cup”, *The Physics Teacher* 48, 292−295 (May 2010)

6. For example, www.metlink.org/secondary/a-level/weather_charts/

7. The scale devised by Admiral Sir Francis Beaufort of the British Navy in 1806 is still used in marine forecasts. The scale is based on the visible effects of wind. For example at a level 6 on the Beaufort scale, larger branches of trees are moving and umbrellas become unwieldy.

8. The equilibrium consideration is only valid if the moving air is not making sharp turns. The meaning of a "sharp turn", however, is not obvious at all. For example, in order to refute the myth of the plug hole, we need to show that the draining water is, indeed, making a sharp turn, which involves an inward net force several order of magnitudes greater than the Coriolis force. The use of algebra alone will not reveal that: it is necessary to compare acceleration components numerically.

9. meted.ucar.edu;

10. weatheronline.co.uk, height 500 hPa ECMWF (gpdm) Sun 29/07/18 00UTC (Fri 00+48)

11. If we wish to stick with isobars instead of considering the topography of pressure surfaces, we can say that in an isobar chart of a little higher altitude, the very same curves are drawn on the map, but each curve will bear a pressure label reduced by the exact same amount. Thus the pressure change per unit horizontal distance will remain the same as at a lower level, and therefore – since density is uniform – the horizontal velocity will also be the same.

12. www.youtube.com/watch?v=1DXHE4kt3Fw

13. https://earthobservatory.nasa.gov/images/987/atmospheric-vortices-near-guadalupe-island

14. Google Earth

15. Seelye Martin, Robert Drucker, “The effect of possible Taylor columns on the summer ice retreat in the Chukchi Sea,” *Journal of Geophysical Research* 102/C5, 10,473−10,482 (May 1997)

16. Jin-Ping Zhao, Jiu-Xin Shu, M. Jin, Chaolun Li, Yu-Tian Jiao, Yong Lu, “Water mass structure of the Chukchi Sea during ice melting period in the Summer of 1999,” *Advances in Earth Science* 25, 154−162 (February 2010).