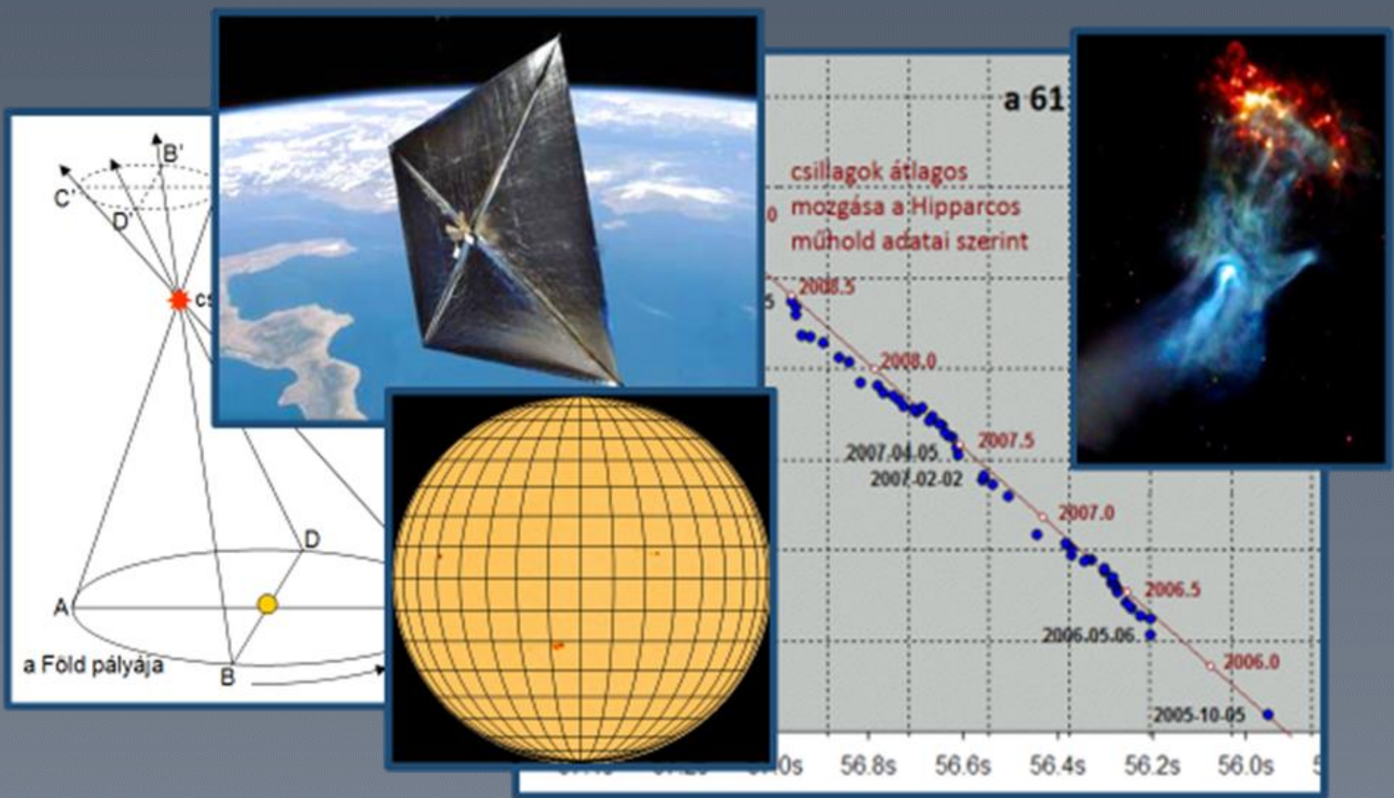
Andrea Gróf

# SPACE PHYSICS

## Selected nonconventional problems on astronomy and space research for high school students




ELTE DOCTORAL SCHOOL OF PHYSICS

**ANDREA GRÓF**

# SPACE PHYSICS

# Selected nonconventional problems on astronomy and space research for high school students

**ELTE DOCTORAL SCHOOL OF PHYSICS BUDAPEST**

**2021**

The publication was supported by the Subject Pedagogical Research Programme of the Hungarian Academy of Sciences

Reviewers:

Zsuzsa Horváth (astronomy), Ákos Szeidemann (didactics)

Translation from Hungarian: Béla Borsos



ELTE Doctoral School of Physics
Responsibe publisher: Dr. Jenő Gubicza
Budapest 2021

Compiled by Andrea Gróf

Reviewer of professional issues: Zsuzsa Horváth
Reviewer of didactics: Ákos Szeidemann

English translation by Béla Borsos

The preparation of the workbook was supported by the Subject Pedagogical Research Programme of the Hungarian Academy of Sciences.
2018

# Table of Contents

# Introduction

The problem book entitled *Exoplanets and spacecraft. Exercises in Astronomy and Space Exploration for high school students* was compiled in 2017, investigating the possibilities to explain modern astronomy to secondary school students. The world of planets in remote stellar systems was introduced, the methods which led to their discovery, the issue of habitability, and we dealt with the findings of the more and more space probes sailing through the Solar system and of the space telescopes, just like the asteroid research which is coming to the foreground recently.

However, not only the most recent achievements of sciences offer novel non-conventional challenges developing the problem solving thinking and shaping the attitudes of students in the field of astronomy and space discovery. It is also important to get astonished at the logical conclusions which could lead the way to then new knowledge and information at the level of development of various ages across the history of technology. The problems in the second collection were selected in the pursuit of this consideration. This way the actual development of technology can also be demonstrated with the help of the problems. Just like in the first selection, special attention was paid to provide an opportunity to process real data, interpret images and graphs taken from true life and for other student activities.

Traditionally, problems dealing with the properties and movements of the celestial bodies appear in physics education. However, it is not customary to touch upon the sources of the qualitative or quantitative information involved in such problems. Even though the fact provides enough food for thought that you can only measure angular distances on the sky directly, you cannot measure absolute distances or depths. One set of problems is therefore based on the conclusions to be drawn from angular measurements. Colourful photographs shot by the space telescope on remote nebulas and the travel of Mercury across the disk of the Sun (which could be observed not so long ago in 2016) are included in the problem book just like a student activity using a pinhole camera for measurements, or the possibilities how images can be recorded. In the case of the International Space Station ISS and the of movements of the sunspots the attention of the students is drawn to the fact that the actually observed movements are generated by the composite of various circular motions.

In an additional chapter historic measurements are carried out with the use of modern technology. Using relatively recent data provided by state-of-the-art instruments we reproduce the way Kepler mapped the orbit of Mercury, and the way Rømer measured the speed of light. The elevation of the mountains on the Moon is measured current imaging technology but following the method used by Galileo, and a sequence of problems demonstrates how Bessel could detect the parallaxes of stars by which he could finally provide the direct evidence that the Earth was orbiting around the Sun. In the course of the problems we emphasize that this result was not only a brilliantly executed interpretation of the observation provided by Bradley (which will make it necessary to learn some new concepts), but also a great technological achievement: Bessel’s observations were repeated between 2005 and 2009 using a telescope similar to Bessel’s, but the data were recorded and processed by computers.

Relationships expressing one quantity as a function of another quantity has always played an important role in physics teaching. In most exercises in physics the nature of this relationship is considered given. However, the data of astronomical measurements provide the opportunity to look for a linear approximation to set up a simple model for complicated relationships, to deal with the physical and geometrical background underlying the non-linear relationship observed, and to investigate the scope and limitations of the relationship established.

Non-conventional exercises can also be obtained when knowledge acquired in different chapters of physics teaching are combined within a single problem. Several problems combine the mechanics and thermodynamics of gases to draw conclusions for the composition of the atmospheres on the different planets as well as to provide a rough estimate of the temperature inside the Sun. From the changes of momentum of the photons you will get to spacecraft accelerated with the help of radiation pressure, a possibility investigated intensively at the time being.

Finally the last set of exercises focuses on the amazing amount of information about stars (temperature, mass, material composition, etc.) in spite of the fact that most of them are merely bright spots on the sky without details, thus all information is derived from the spectral distribution of the light arriving from them. The theoretical background of these exercises extends somewhat beyond the scope of conventional secondary school physics, since it includes the laws of blackbody radiation (Wien's displacement law and the Stefan−Boltzmann law). Since these laws are included in the training of secondary school teachers, it only requires a little extra work to guide students from the examination of the characteristics of stars to the study of surface temperature of planets and the role of the atmospheric greenhouse gas effect.

# 1. Angular measurements on the sky

## DETERMINATION OF DIMENSIONS WHEN DISTANCE IS KNOWN

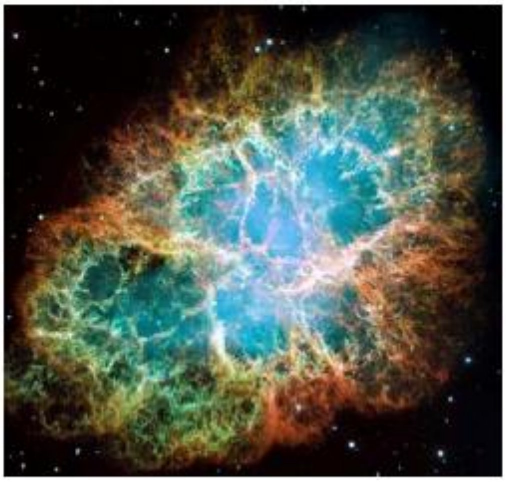
**1.1** The X-ray emanating nebula in the picture, reminiscent of a human hand (it is also called the hand of God nebula) was created together with another, very dense object with a diameter of only20 km. In the middle of the image taken by the Chandra space telescope, behind the thumb there is a young pulsar, a rapidly revolving neutron star marked PSR B1509-58, which emits a huge amount of energy.
According to astronomers the pulsar and the nebula are approximately 1700 years old, and are situated at a distance of about 17 000 light years from us.
The structures stretching our towards the right side that look like fingers indicate a high energy flow of particles, probably carrying energy to the neighbouring RCW 89 gas nebula, triggering a strong radiation of the material of the nebula in the X ray spectrum (these are the red and orange nodes at the top right of the image).
(a) The width of the picture is 19 minutes of arc. How much is this width in light years?

(b) How many light years is the distance between the bright spot in the "thumb" of the hand, that is, the pulsar, and the ring of light nodes of RCW 89?

(c) If the velocity of the interstellar gas is 10 000 km/s, how much time is needed for flow of material to reach RCW 89?

http://spacemath.gsfc.nasa.gov

**1.2** (a) The picture taken by the Hubble-space telescope (2005), shows the Cancer nebula. One millimetre on the picture corresponds to 0.2 light years. The nebula was created by a supernova explosion observed back in 1054, and the gas cloud keeps on expanding ever since. What is the average speed of expansion expressed in m/s?



http://spacemath.gsfc.nasa.gov

The pulsar is a rapidly rotating neutron star remaining after a supernova explosion, which emits a beam of radio radiation at a frequency equal to its speed of rotation. The Cancer pulsar found in the constellation Cancer is a remnant of the supernova explosion observed in 1054. The period of the radio radiation is currently 0.033 s, but it keeps on increasing continually, as accurate measurements indicate. The growth rate is $1.26 \cdot 10^{-5}$ s each year.

(b) How much is the rotational acceleration of the pulsar?

(c) Assuming that the rotational acceleration is constant, in how many years would the rotation stop?

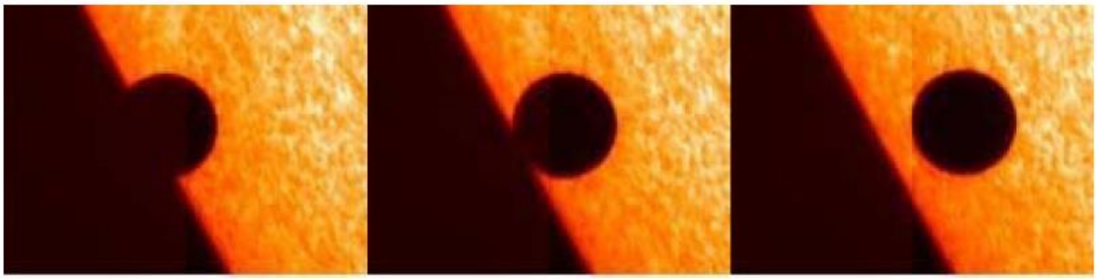
(d) Assuming that the rotational acceleration is constant, what was the period at the birth of the pulsar?

# 1. Angular measurements on the sky

## ANGULAR SIZE OF THE DISKS OF THE SUN AND OF THE MOON

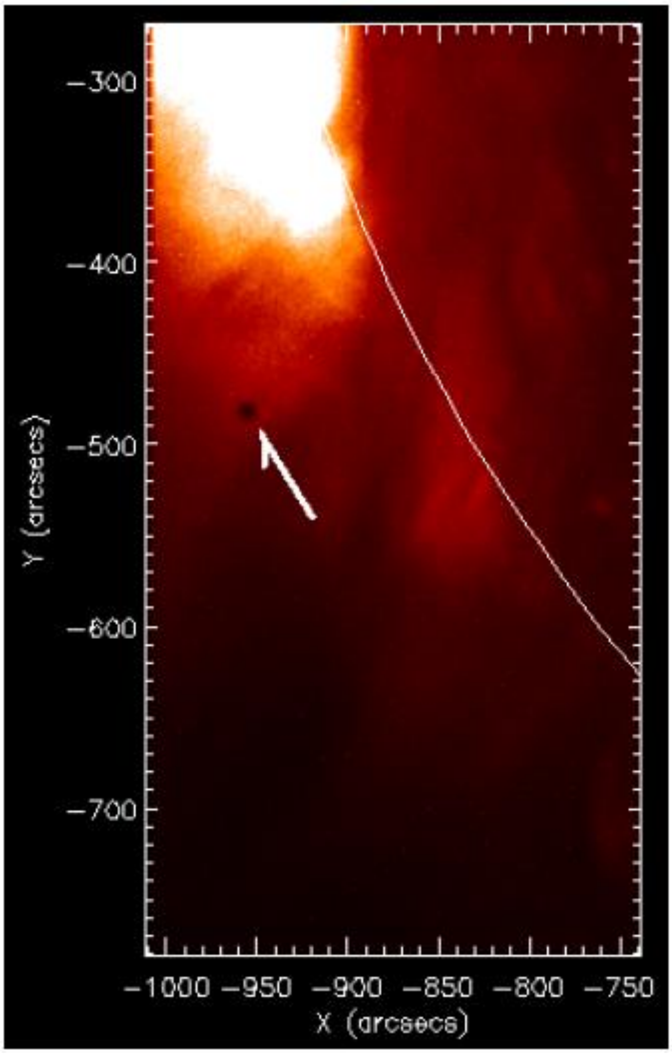
**1.3** As seen from the Earth, Mercury makes a transit in front of the Sun once or twice in a decade. (Such an event occurred on 9 May 2016.)

(a) Why are such transitions so rare?



Solar Optical Telescope (SOT) http://spacemath.gsfc.nasa.gov

(b) The image below was taken by the satellite Hinode during the 8 November 2006 transit. The white line is the edge of the disk of the Sun and the arrow shows Mercury travelling in front of the solar corona. At the time of the passage the angular diameter of Mercury was 10 angular seconds. What was the angular diameter of the Sun according to the Hinode EIS image?




EUV Imaging Spectrometer (EIS). http://spacemath.gsfc.nasa.gov

**1.4** Prepare a pinhole camera from a tube at least half a metre long: (It can be assembled from multiple pieces, you can use for instance some paper cylinders found in all households.) With dark coloured adhesive tape, attach a disk cut out of squared graph paper to one end of the tube, and a disk of opaque cardboard to the other end. Pierce the middle of the cardboard disk using a needle.

Direct the hole towards the Sun so that the disk of the Sun should appear on the screen made of graph paper. (If your hands are shaking, lean the tube against the back of a chair.) It might be necessary to cover up your head and the squared paper end of the tube with a dark blanket like photographers did in old times so that no direct sunshine will blind you.

Read the diameter of the image of the disk of the Sun from the graph paper. Measure the length of the tube and determine the angular diameter of the Sun (that is, the angle subtended by the Sun as seen from the Earth).

**1.5** The angular diameter of the Moon varies between 29’22’’ and 33’31’’. What may be the focal length of the telescope objective if you want to fit the image of the Moon on a standard 22 × 16 mm CCD chip at all times?

**1.6** In order to observe sunspots with a telescope the image created by the telescope is projected on a screen placed behind the eyepiece lens. The focal length of the objective is 120 cm, that of the eyepiece is 2.0 cm. At what distance from the eyepiece do you have to put the screen order to get an image of the Sun with a diameter of 16 cm?

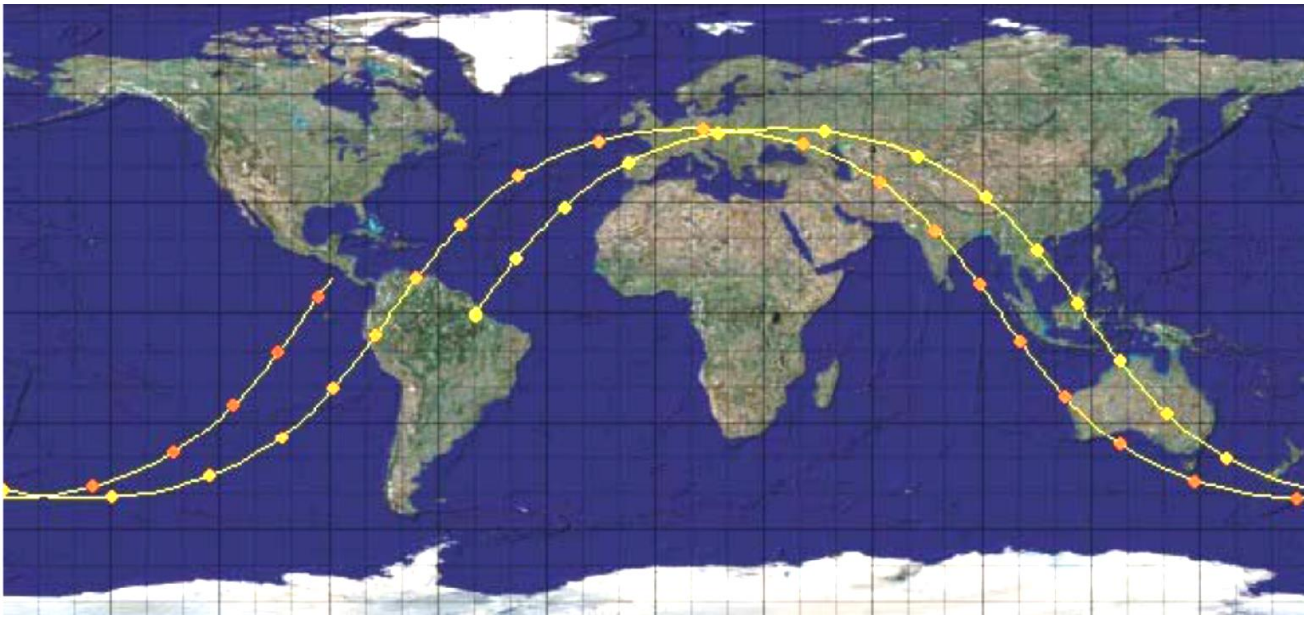
# 1. Angular measurements on the sky
## ORBITING AND ROTATION ABOUT AN AXIS

**1.7** The orbital period of the International Space Station (ISS) around the Earth is 90 minutes. While the ISS circles around the Earth, the Earth also turns around underneath. This means that the geographic longitude of the ISS keeps changing all the time. The lower figure shows the position of the ISS projected onto the surface of the Earth during the time of two orbiting periods at 5 minute intervals. The Equator runs in the middle of the image horizontally, the side of each square is 10 degrees.

http://spacemath.gsfc.nasa.gov

(a) Through what angle does the Earth turn during one period of the ISS?

(b) How many sunrises and sunsets may the crew of the space station observe during a 24 hour earth day?



http://spacemath.gsfc.nasa.gov

**1.8** The space probe Discoverer II orbited above the surface of the Earth at a height of approximately $6.67 \cdot 10^3$ m, in a nearly circular orbit such that it passed over both poles. Suppose it passed above Budapest during a certain occasion of crossing Europe. Where did it cross Europe the following time?

**1.9** (a) What is the ratio of the orbital period of Mercury to the period of rotation about its axis?

Note:
Due to its small size (its angle of vision is 13 angular seconds at most) and its proximity to the Sun it is very difficult to observe any details on the surface of Mercury. Based on observations dated from the 19th century it was thought that Mercury turned the same side to the Sun all the time. This would not have been very surprising, it would mean that the rotation of Mercury has been synchronised in a 1:1 resonance with the orbiting of the planet due to the tidal forces in the neighbourhood of huge mass of the Sun, just as it happened in the case of the Moon orbiting the Earth. In 1965 however, the Doppler shift of signals reflecting from Mercury were investigated using the Arecibo radio telescope operated in Puerto Rico, and it was found that its rotational period was 58.65 days, a duration which does not coincide with the 87.97 days orbiting period. The result was confirmed by the data obtained from the Mariner space probe in 1974.

(b) Assume that the orbit of Mercury is circular. How long is a solar day on Mercury (in other words, how much time elapse between two subsequent sunrises or sunsets)?

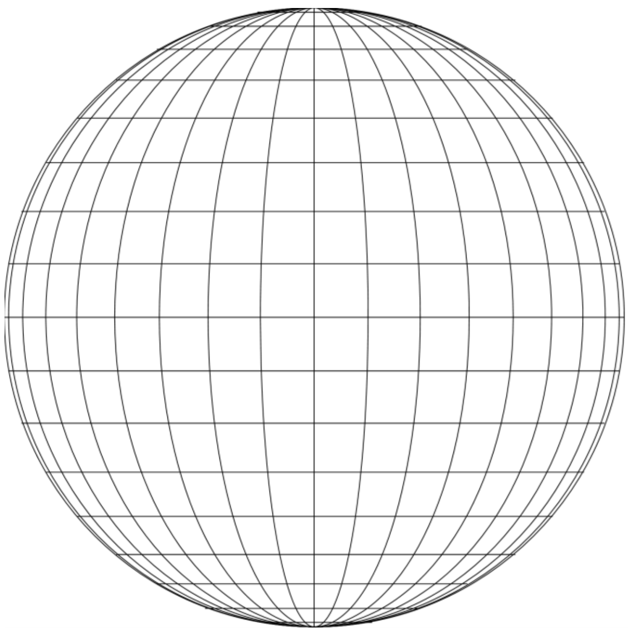
(c) In fact, the orbit of Mercury is not a circle, is a relatively elongated ellipse. At perihelion the orbital speed of Mercury is greater, the angular speed of orbiting reaches the value of $1.26\cdot10^{-6}$/s. Compare this maximum angular speed with the angular speed of the rotation of Mercury about its axis. As a consequence of this result, what is the apparent motion of the Sun on the sky for an observer on Mercury?

**1.10** A következő képsorozat a napfoltok helyzetét mutatja a feltüntetett időpontokban. A képeken (Forrás: soho_realtime_hmi_Continuum) végzett mérések segítségével határozd meg a napfoltok körülfordulási idejét két különböző heliografikus szélességen.

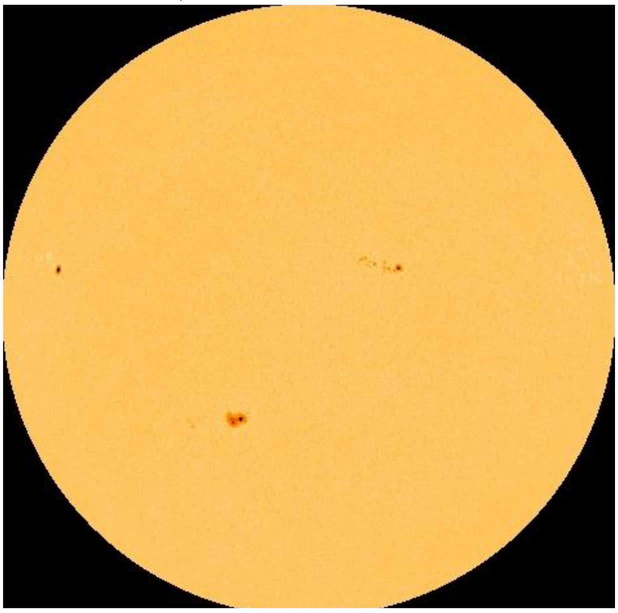
A mérést megkönnyítheti az alábbi koordinátahálózat. (Ha a fehér színt átlátszóvá tesszük, megfelelő méretben ráhelyezhető a többi képre.)

**1.10** The following series of images shows the position of sunspots at the dates and times indicated. Determine the rotation period of the sunspots at two different heliographic latitudes by measurements carried out on the images (Source: soho_realtime_hmi_Continuum).

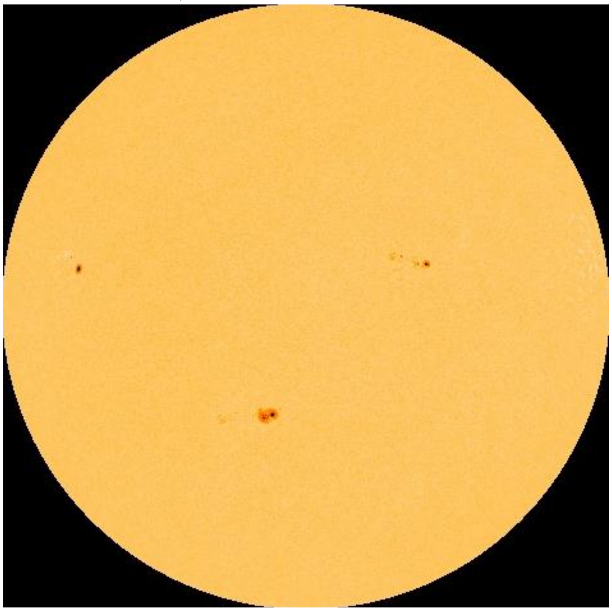
The measurement can be made easier by the coordinate grid shown below. (When white is rendered transparent, it can be placed onto the other photos.)



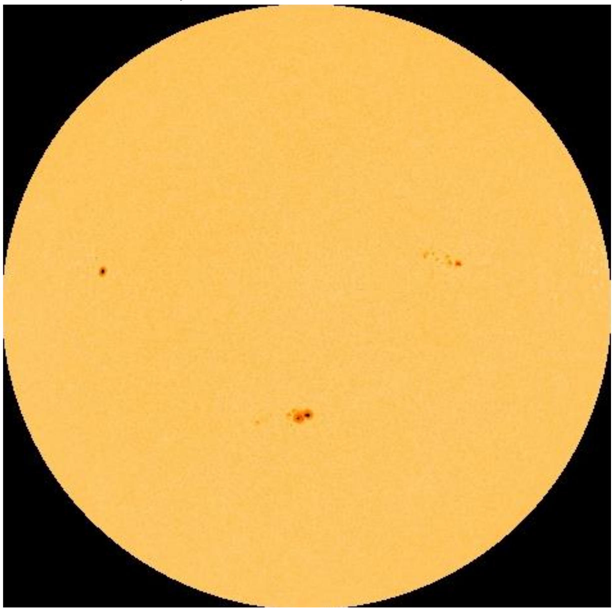
**1.** 08.10.2016, 07:30



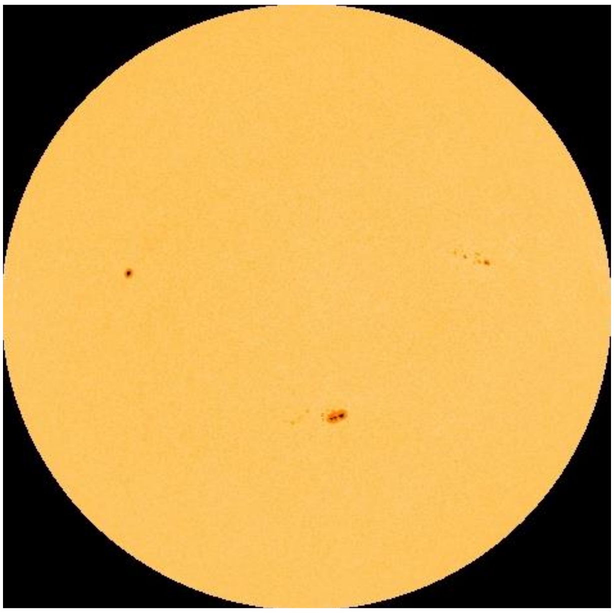
**2.** 08.10.2016, 19:30



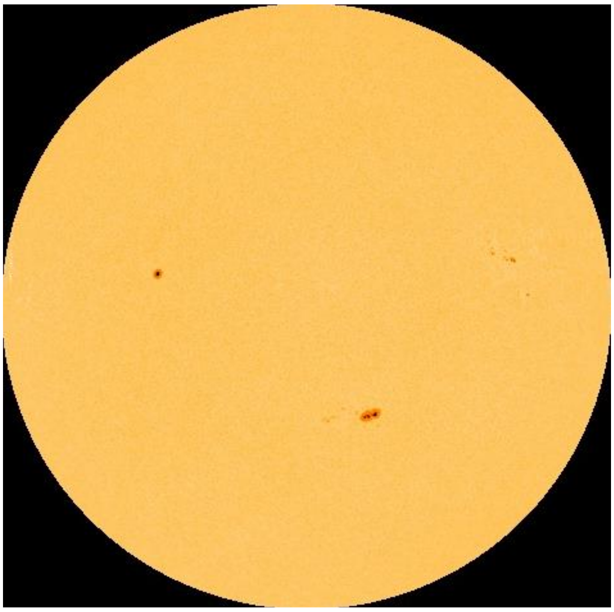
**3.** 09.10.2016, 07:30



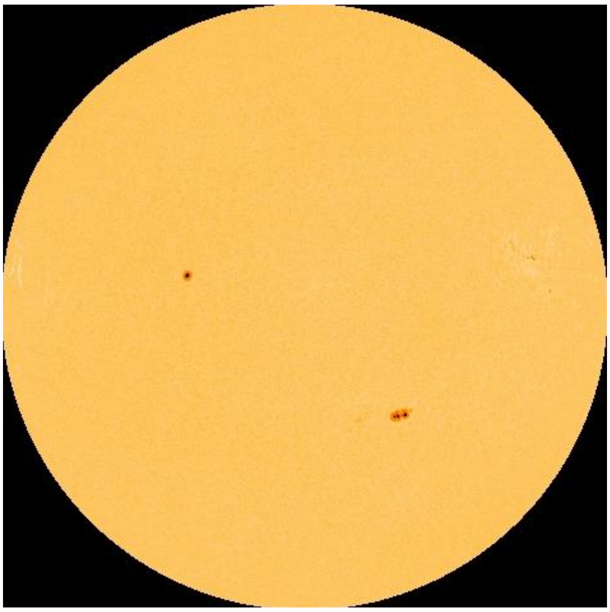
**4.** 09.10.2016, 19:30



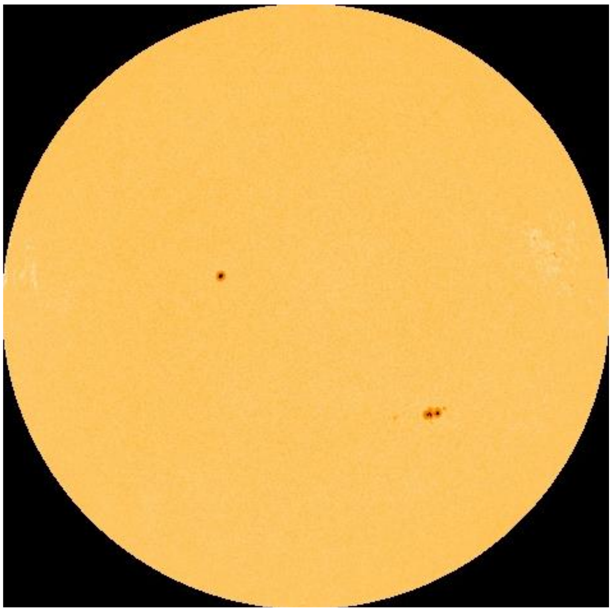
**5.** 10.10.2016. 07:30



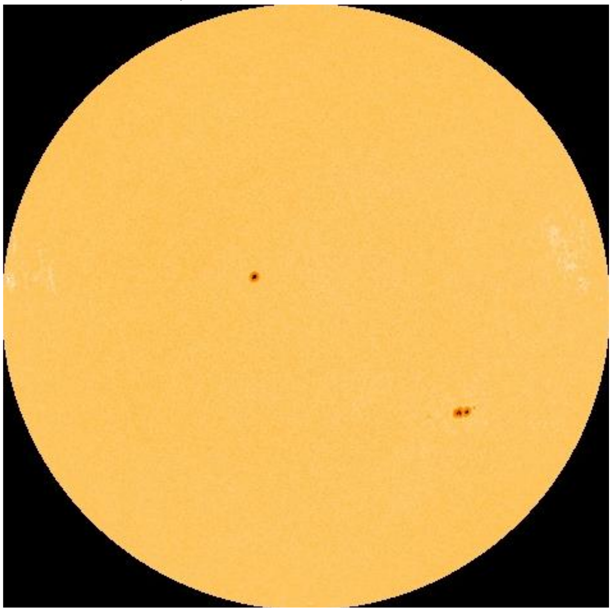
**6.** 10.10.2016. 19:30



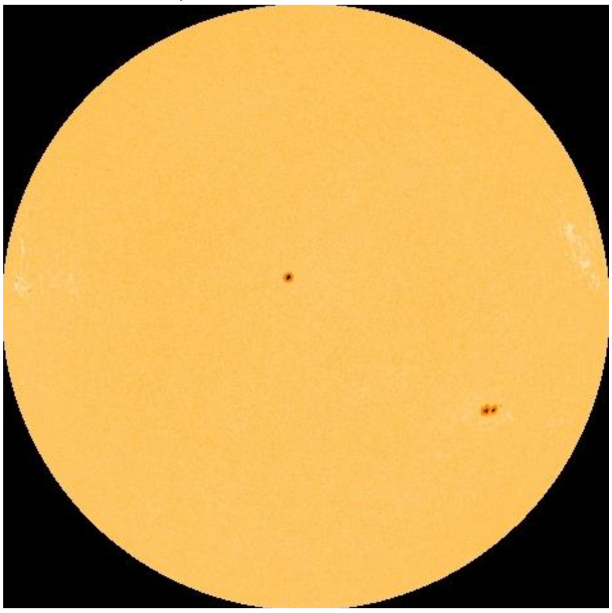
**7.** 11.10.2016, 07:30



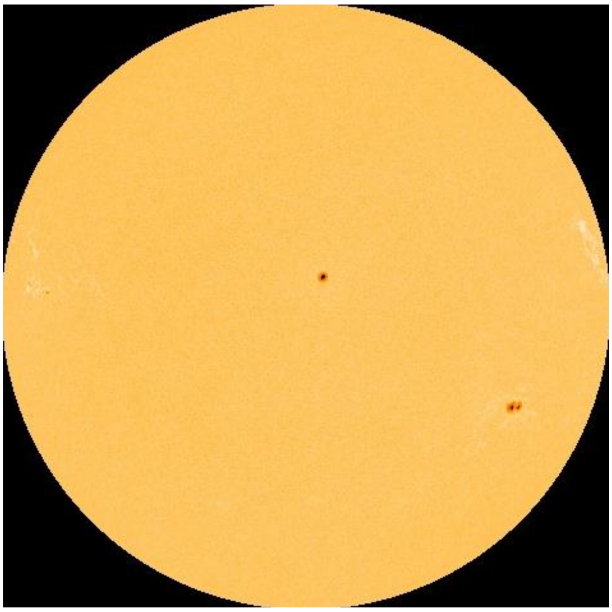
**8.** 11.10.2016, 19:30



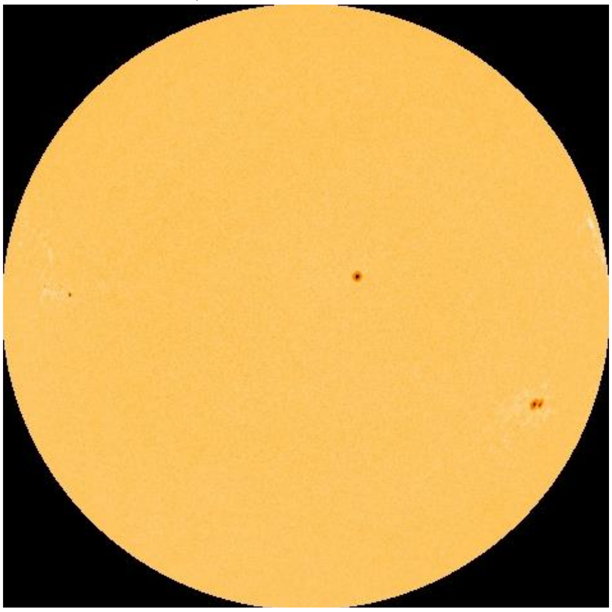
**9.** 12.10.2016, 07:30



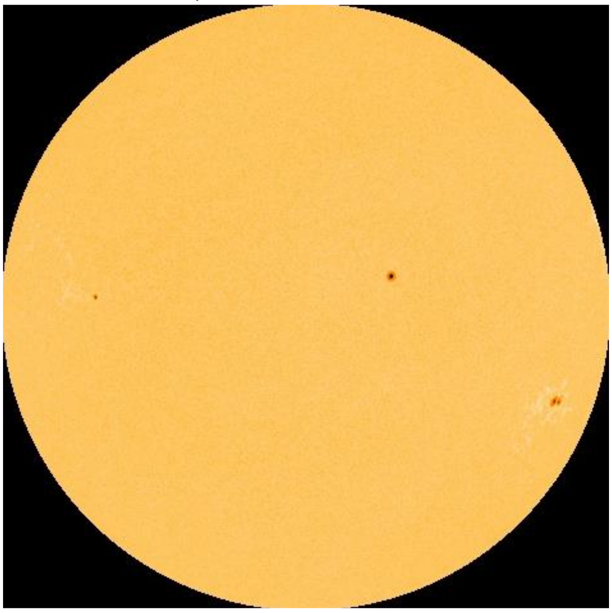
**10.** 12.10.2016, 19:30



**11.** 13.10.2016, 07:30



**12.** 13.10.2016, 19:30



# Solution 1.

**1.1** (a) 19’ = 19/60 = 0.32° = $5.5\cdot10^{-3}$ rad.

$17\,000\cdot5.5\cdot10^{-3}$ = 94 light years

(b) If the picture is 19 units wide, the distance in question will be about 9 units (measured with a ruler).

94·9/19 = 45 light years.

(c) 1 light year = $9.5\cdot10^{12}$ km.

45 light years = $4.3\cdot10^{14}$ km

$(4.3\cdot10^{14})/10\,000 = 4.3\cdot10^{10}$ s = 1400 years

**1.2** (a) The largest distance across the nebula in the image is about 6.5 cm, that is 65·0.2 = 13 ly. Radius is half of this. About 950 years elapsed. The speed is 0.5·13 light years / 950 years =

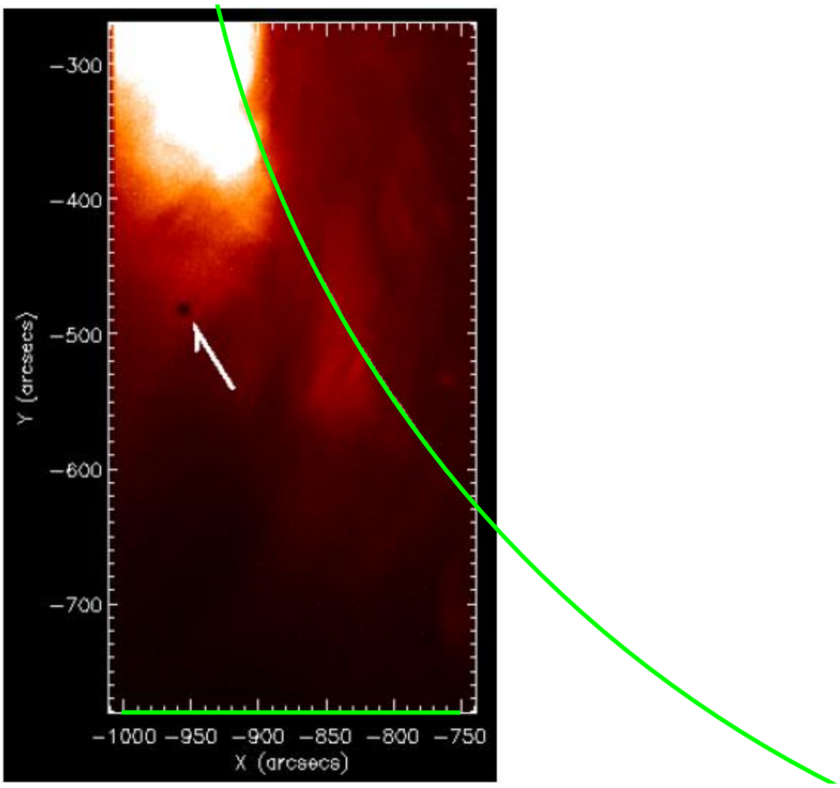
= $0.5\cdot3.0\cdot10^{8}\cdot13/950 = 2.1\cdot10^{6}$ m/s.

(b) $\omega_0 = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.033} = 190\,\text{rad/s}$

$$\Delta\omega = \omega - \omega_0 = \frac{2\pi}{T+\Delta T} - \frac{2\pi}{T} =$$

$$= \frac{2\pi}{0{,}033 + 1{,}26\cdot10^{-5}} - \frac{2\pi}{0{,}033} = -0{,}0727\ \text{rad/s}$$

$$\beta = \frac{\Delta\omega}{\Delta t} = \frac{-0{,}0727}{365\cdot24\cdot3600} = -2{,}3\cdot10^{-9}\,\text{rad/s}^2$$




Or:

$$\beta = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{2\pi}{T}\right) = -\frac{2\pi}{T^2}\cdot\frac{dT}{dt} =$$

$$= -\frac{2\pi}{0{,}033^2}\cdot\frac{1{,}26\cdot10^{-5}}{365\cdot24\cdot3600} = -2{,}3\cdot10^{-9}\,\text{rad/s}^2$$

(c) $t = \dfrac{-\omega}{\beta} = \dfrac{-2\pi}{\beta T} = \dfrac{2\pi}{2{,}3\cdot10^{-9}\cdot0{,}033} =$

$=8.3\cdot10^{10}$ s = 2600 years.

(d) The time elapsed is about 1000 years (no need to calculate more precisely).

$$\Delta\omega = \beta\cdot t = -2{,}3\cdot10^{-9}\cdot1000\cdot365\cdot24\cdot3600 \approx$$

$$\approx -70\,\text{rad/s}$$

The initial angular velocity is estimated to be

190 + 70 = 260 rad/s

$$T = \frac{2\pi}{260} = 0{,}024\text{s}$$

**1.3** (a) Mercury and the Earth do not orbit the Sun in exactly the same plane. A transit can only be seen when both planets are on the line of intersection of the orbital planes.

(b) If the 250’’ measured on the picture along the horizontal axis is 8.3 cm long, the diameter of the circle will be 58 cm, corresponding to 250·58/8.3 = 1800 arc seconds.

**1.4** For example, three cylindrical potato chips boxes will make a tube of length 750 mm. Since the subtended angle is small, an image diameter of (7 ± 0.5) mm, expressed in radian will be

$$\frac{7}{752} = 9{,}3\cdot10^{-3}\,\text{rad} =$$

$$= 0{,}53^\circ \pm 7\% =$$

$$= (0{,}53 \pm 0{,}04)^\circ.$$

**1.5** For such a distant object the image distance will be equal to the focal length. The size of the object and the image is proportional to the distances:

$$\frac{h'}{f} = \frac{h}{t}$$

$\frac{h}{t}$ is the angular diameter of the Moon, which is at most 33’31’’ = 0.00975 rad. The smaller dimension of the chip is 16 mm, this is the largest possible diameter of the Moon image.

$$h' = \frac{h}{t} \cdot f = 0{,}00975 f$$

This has to fit the on chip: $0{,}00975 f \le 16$

$$f \le 1640 \text{ mm.}$$

**1.6** The distance of the Sun is $1.5 \cdot 10^{11}$ m, its diameter is $1.4 \cdot 10^{9}$ m. An image of size

$$1{,}4 \cdot 10^{9} \cdot \frac{1{,}2}{1{,}5 \cdot 10^{11}} = 1{,}12 \text{cm}$$

forms in the focal plane of the objective. This is the object for the eyepiece, therefore the magnification of the eyepiece is

$$\frac{16}{1{,}12} = 14{,}3 \cdot$$

$$\frac{1}{f} = \frac{1}{t} + \frac{1}{k}$$

$$\frac{1}{2} = \frac{1}{t} + \frac{1}{14{,}3t}$$

$$\frac{15{,}3}{14{,}3t} = \frac{1}{2}$$

$$k = 14{,}3t = 2 \cdot 15{,}3 = 30{,}6 \text{ cm}.$$

The screen must be put in front of the eyepiece at this distance.

**1.7** (a) 90 minutes = 1.5 hours. The rotation period of the Earth is 23.93 hours,

$$\frac{1{,}5}{23{,}93} \cdot 360° = 22{,}6° \cdot$$

(This can also be read from the diagram, though very imprecisely.)

(b) The time spent between two subsequent sunsets or sunrises will be more than 1.5 hours since the Earth turns around the Sun:

$$\frac{1}{T} = \frac{1}{1{,}5} - \frac{1}{365{,}26 \cdot 24}$$

$T$ = 1.5002 hours.

The difference is about 1 sec. During 24 hours

$$\frac{24}{1{,}5002} = 15{,}997$$

orbital periods will take place, almost 16. This will nearly always imply 16 sunrises and 16 sunsets. (About 45 minutes after sunrise, a sunset occurs.) The 24-hour period, however, can be chosen so that it contain only 15 of one kind of event.

**1.8** The radius of the orbit is

$$r = r + h = 6370 + 7 = 6377 \text{km}.$$

The orbital period is $T = 2\pi\sqrt{\frac{r^3}{\gamma M}} =$

$$= 2\pi\sqrt{\frac{(6{,}377 \cdot 10^6)^3}{6{,}67 \cdot 10^{-11} \cdot 5{,}98 \cdot 10^{24}}} = 5070 \text{s}.$$

During this time the Earth turns 21° from west to the east, thus the probe passes 21° to the west from Budapest (longitude E19°), at longitude W2°, that is (above England, approximately at Birmingham).

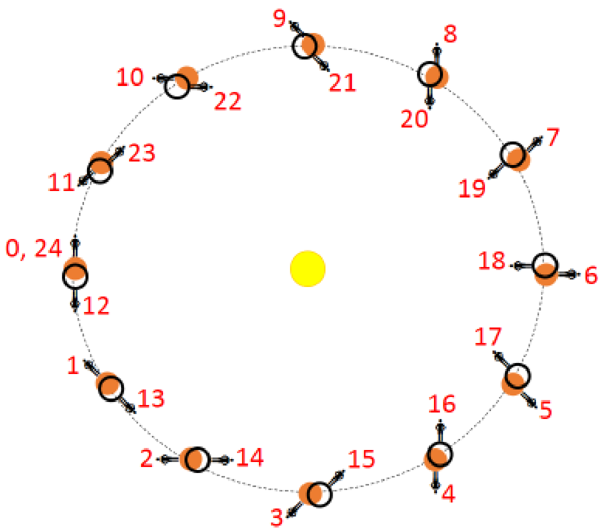
**1.9** (a) 87.97 days/58.65 days = 1.500. That is, they are in a 3:2 resonance.

(b) $\frac{1}{58{,}65} - \frac{1}{87{,}97} = 0{,}005683$

1/0.005683 = 1760 days,

that is two Mercury-years.

The figure below shows the observer standing on the surface of Mercury through two orbital periods (i.e. three rotations about the axis), at equal intervals. The observer will experience sunsets in positions 0 and 24, and a sunrise in position 12.

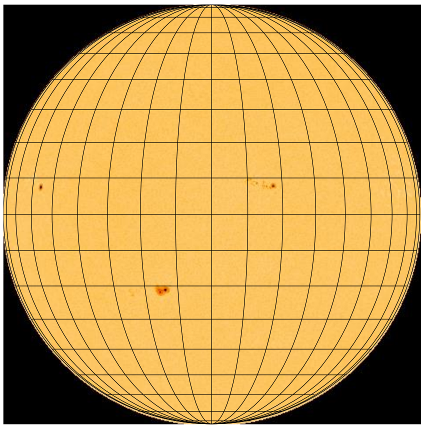


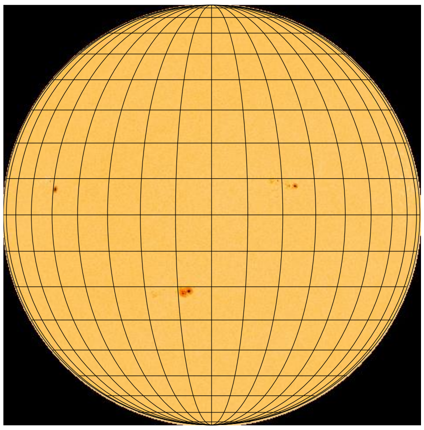

(c) The angular velocity of rotation about the axis is

$$\frac{2\pi}{58{,}464 \cdot 24 \cdot 3600} = 1{,}24 \cdot 10^{-6} \text{ /s}.$$

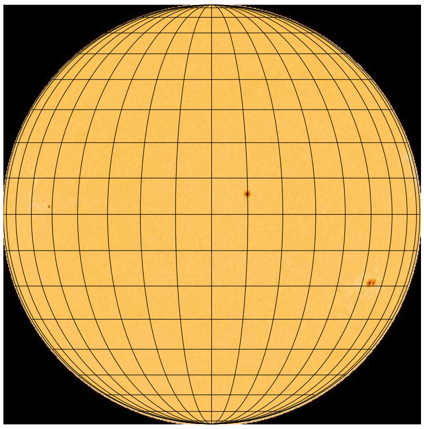
The angular velocity of orbiting is larger, so, as seen from Mercury at perihelion, the Sun would move backwards on the sky, from west to east for a while. For the observer staying at the right place on the planet this may also mean that the sun rises, then it sets again (in the east!) and rises anew.

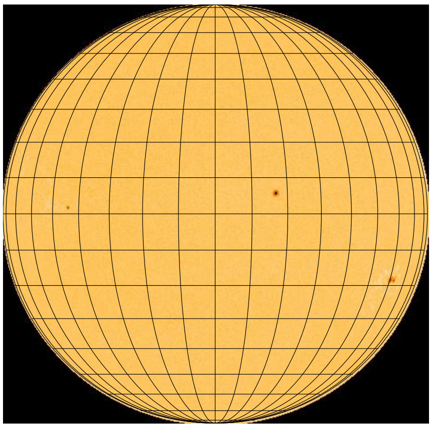
**1.10** By laying the coordinate grid over the images, the motion of the two sunspot groups can be followed for a period of five and a half days.





and so on …





One spot is located at a latitude of about 7°.
Its longitude changed uniformly from –56° to +10° during 5.5 days, that is, it turns through 66°. Take into account that the observation is made from the Earth, which orbits the Sun in the same direction as the rotation of the Sun. It turns 360° in one year, that is, 1° in a day, and 5.5° during 5.5 days.

During 5.5 days the spot would turn around the axis of the Sun through

66 + 5.5 = 71.5°.

Using direct proportionality, 28 days will be needed to cover 360°.

During the same time the other spot, at a latitude of approximately –20° will get from longitude –14° to +50°.

64 + 5.5 = 69.5°

Thus its turnaround time will be 29 days.

Or:
One spots turns 66° in 5.5 days as seen from the Earth, this means a period of

$T' = 5.5 \cdot 360/66 = 30$ days.

The rotational angular velocity experienced by an Earth observer is less than the true angular velocity because the Earth orbits around the Sun in the same direction as the rotation. The angular velocity observed from Earth is the resultant of the rotation of the sunspot and the orbiting of the Earth, in other words it is the difference of the two angular velocities:

$$\frac{2\pi}{30} = \frac{2\pi}{T} - \frac{2\pi}{365{,}25}$$

$$\frac{1}{T} = \frac{1}{30} + \frac{1}{365{,}25}$$

$T = 28$ days

A tengely körüli forgás periódusára

$$\frac{1}{T} = \frac{1}{30{,}9} + \frac{1}{365{,}25}$$

$T = 29$ nap

The other spot turns 64° in 5.5 days as seen from Earth, meaning a period of

$T' = 5.5 \cdot 360/64 = 30.9$ days.

The rotation period is

$$\frac{1}{T} = \frac{1}{30{,}9} + \frac{1}{365{,}25}$$

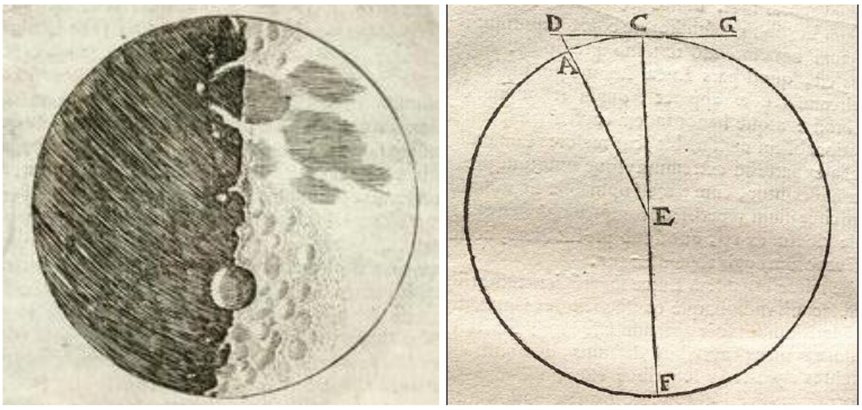
$T = 29$ days

# 2. Old measurements using modern data and technology

## THE HEIGHT OF THE MOUNTAINS ON THE MOON: GALILEO'S MEASUREMENTS

**2.1** (a) The longer the shadows cast by the surface features of the Moon the better they can be distinguished. Which phase of the Moon provides the best conditions to observe the Moon in the evening hours?

(b) In his book Sidereus Nuncius (1610) Galileo estimated the height of the Moon's mountains. The figure drawn by him shows light still touching the peak of the mountains standing close to the boundary of the illuminated half of the Moon (the terminator line). He determined that one of such illuminated peaks was seen at a distance of 1/20 moon-diameter from the terminator. Later on the radius of the Moon was found to be 1740 km. Use this information to determine the height of the mountain.

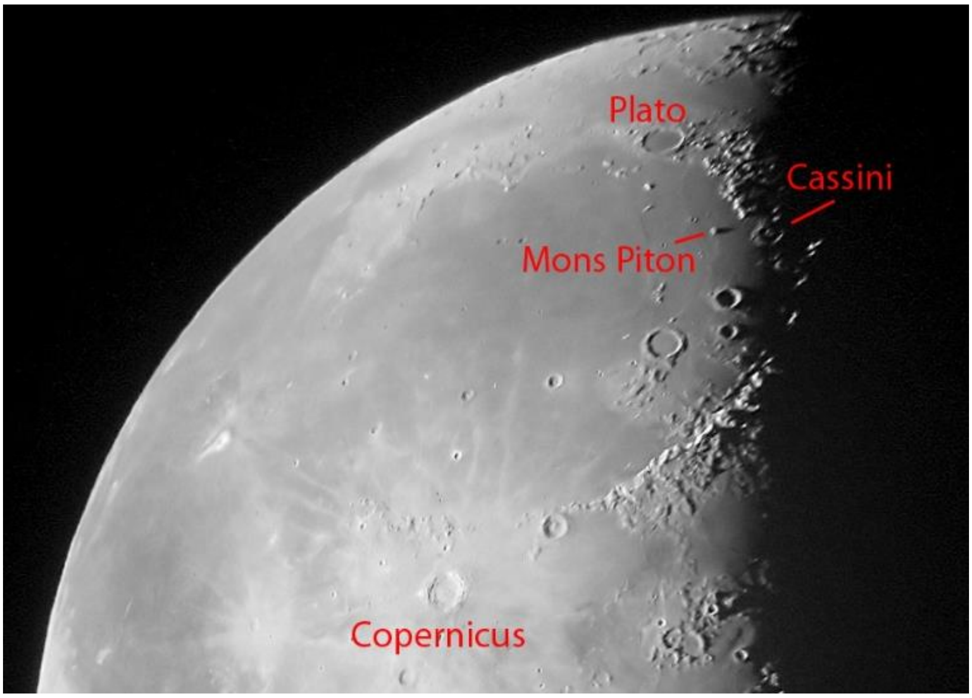



Galileo's drawings


SIDEREVS
NVNCIVS
MAGNA, LONGEQVE ADMIRABILIA
GALILEO GALILEO
PATRITIO FLORENTINO
PERSPICILLI
QVATVOR PLANETIS
MEDICEA SIDERA
NVNCVPANDOS DECREVIT.
VENETIIS, Apud Thomam Baglionum. M DC X.

The title page of Sidereus Nuncius

**2.2** Using the known radius of the Moon and modern photographic technology, the height of mountains (or craters) can also be determined from the shadows cast by these topographic features. The image below, taken at last quarter, shows the solitary peak Mons Piton protruding from Mare Imbrium (Sea of Rains). Measure the distance of the mountain from the terminator line and the shadow of the mountain. Determine the height of the mountain from these data.




http://www.jb.man.ac.uk/astronomy/nightsky/Piton1.jpg

# 2. Old measurements using modern data and technology

## THE ORBIT OF MERCURY: KEPLER'S METHOD

**2.3** Kepler recognised the laws of planetary motion when studying the orbit of Mars. Then he extended his examinations to the orbits of the other planets. He used maximum eastern and western elongation data[1] similar to those below to map the orbit of Mercury.

The table shows the values of the largest eastern and western elongations of Mercury measured during three consecutive years. (Kepler had data of about 20 years, recorded by Tycho Brahe.)

| | |
|---|---|
| **2014.** | |
| January 31 | E18.4° |
| March 14 | W27.6° |
| May 25 | E22.7° |
| July 12 | W20.9° |
| September 21 | E26.4° |
| November 1 | W18.7° |
| **2015** | |
| January 14 | E18.9° |
| February 24 | W26.7° |
| May 7 | E21.2° |
| June 24 | W22.5° |
| September | E27.1° |
| October 16 | W18.1° |
| December 29 | E19.7° |
| **2016** | |
| February 7 | W25.6° |
| April 18 | E19.9° |
| June 5 | W24.2° |
| August 16 | E27.4° |
| September 28 | W17.9° |
| December 11 | E20.8° |

Print out the figure below showing the Earth's orbit (considered approximately circular) with the Sun at the centre. Starting from the positions of the Earth corresponding to each of the dates in the table, draw the straight line pointing towards Mercury using a thin-point pencil and a protractor. (In the case of eastern and western elongations, respectively, measure the angle to the left and to the right from the Earth-Sun line, looking from Earth towards the Sun.)

(a) What is the approximate distance in kilometres represented in the drawing by the thickness of the pencil line?

(b) As you have drawn in more and more lines, the orbit of Mercury starts to take shape. Draw it in.

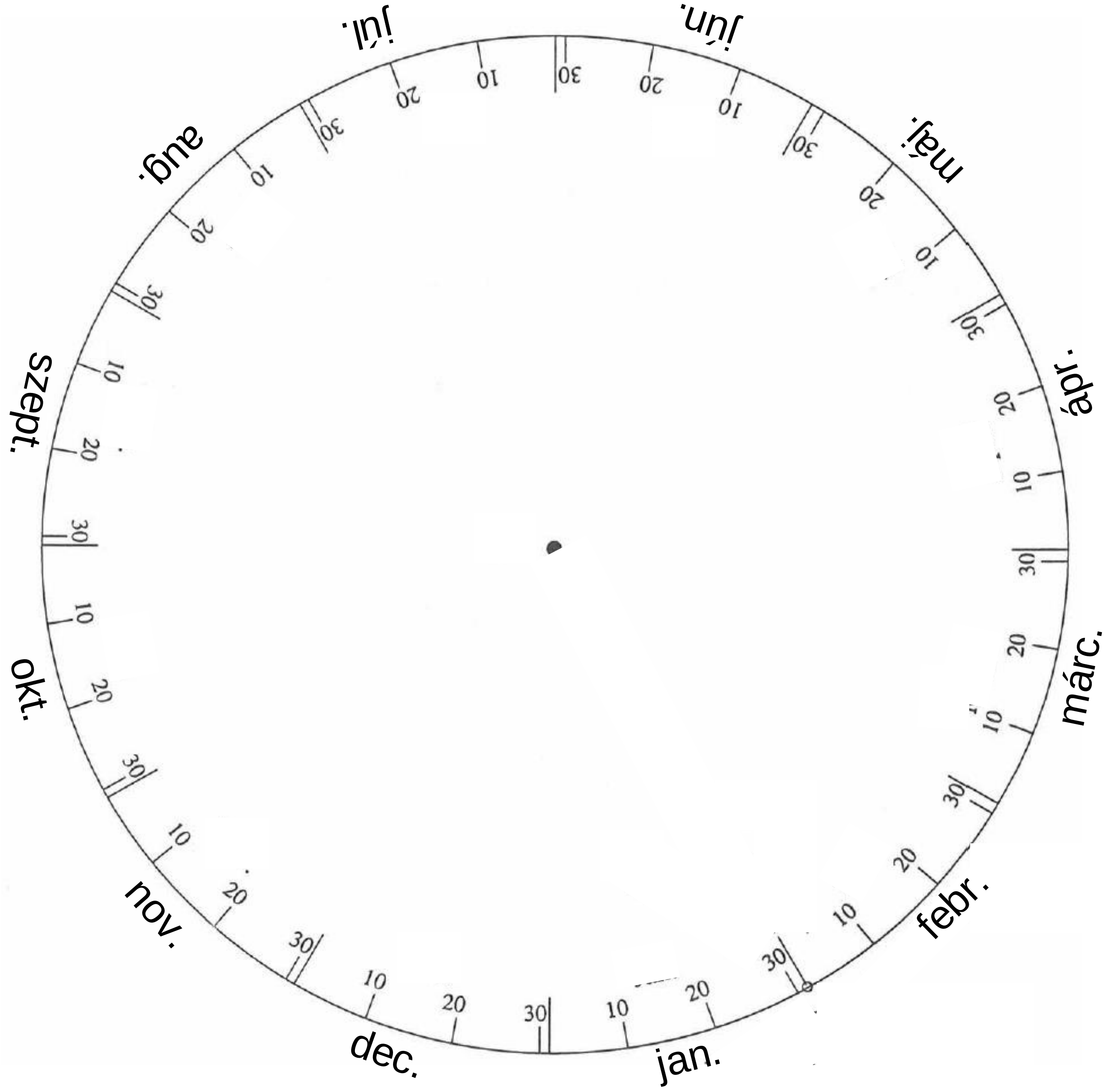
(c) Measure the major axis of the orbit of Mercury on the figure. How many astronomical units is the semimajor axis ($a$)? Compare your result with literature data.

(d) Use the third law of Kepler to calculate the orbital period. Compare your result with literature data.

(e) Let $c$ be the distance of the Sun (focus) from the centre. Determine the ratio $c/a$: this is the eccentricity of the orbit. )? Compare your result with literature data.

---

[1] The position of the inner planets can be characterised by the angle included by the direction of vision with the direction of the Sun (elongation). A maximum elongation exists when the triangle constituted by the Earth, the Sun and a planet is right angled at the planet. The planet is then seen from Earth in a „half" phase.


jan.
febr.
márc.
ápr.
máj.
jún.
júl.
aug.
szept.
okt.
nov.
dec.

# 2. Old measurements using modern data and technology

SPEED OF LIGHT: RØMER'S MEASUREMENT

**2.4** The table shows the dates and times (to the nearest minute, Greenwich winter time) when the innermost moon of Jupiter named Io enters and exits the shadow of Jupiter, respectively, in the course of the year 2017.

(https://www.projectpluto.com/jevent.htm)

January

| | | |
|---|---|---|
| 2 | 00:07 | in |
| 3 | 18:35 | in |
| 5 | 13:03 | in |
| 7 | 07:31 | in |
| 9 | 02:00 | in |
| 10 | 20:28 | in |
| 12 | 14:56 | in |
| 14 | 09:24 | in |
| 16 | 03:53 | in |
| 17 | 22:21 | in |
| 19 | 16:49 | in |
| 21 | 11:17 | in |
| 23 | 05:46 | in |
| 25 | 00:14 | in |
| 26 | 18:42 | in |
| 28 | 13:10 | in |
| 30 | 07:39 | in |

February

| | | |
|---|---|---|
| 1 | 02:07 | in |
| 2 | 20:35 | in |
| 4 | 15:03 | in |
| 6 | 09:32 | in |
| 8 | 04:00 | in |
| 9 | 22:28 | in |
| 11 | 16:56 | in |
| 13 | 11:25 | in |
| 15 | 05:53 | in |
| 17 | 00:21 | in |
| 18 | 18:50 | in |
| 20 | 13:18 | in |
| 22 | 07:46 | in |
| 24 | 02:15 | in |
| 25 | 20:43 | in |
| 27 | 15:11 | in |

March

| | | |
|---|---|---|
| 1 | 09:40 | in |
| 3 | 04:08 | in |
| 4 | 22:37 | in |
| 6 | 17:05 | in |
| 8 | 11:33 | in |
| 10 | 06:02 | in |
| 12 | 00:30 | in |
| 13 | 18:59 | in |
| 15 | 13:27 | in |
| 17 | 07:55 | in |
| 19 | 02:24 | in |
| 20 | 20:52 | in |
| 22 | 15:21 | in |
| 24 | 09:49 | in |
| 26 | 04:18 | in |
| 27 | 22:46 | in |
| 29 | 17:15 | in |
| 31 | 11:43 | in |

April

| | | |
|---|---|---|
| 2 | 06:12 | in |
| 4 | 00:40 | in |
| 5 | 19:09 | in |
| 7 | 13:37 | in |
| 7 | 15:49 | out |
| 9 | 10:18 | out |
| 11 | 04:46 | out |
| 12 | 23:15 | out |
| 14 | 17:43 | out |
| 16 | 12:12 | out |
| 18 | 06:41 | out |
| 20 | 01:09 | out |
| 21 | 19:38 | out |
| 23 | 14:06 | out |
| 25 | 08:35 | out |
| 27 | 03:04 | out |
| 28 | 21:32 | out |
| 30 | 16:01 | out |

May

| | | |
|---|---|---|
| 2 | 10:30 | out |
| 4 | 04:58 | out |
| 5 | 23:27 | out |
| 7 | 17:56 | out |
| 9 | 12:25 | out |
| 11 | 06:53 | out |
| 13 | 01:22 | out |
| 14 | 19:51 | out |
| 16 | 14:19 | out |
| 18 | 08:48 | out |
| 20 | 03:17 | out |
| 21 | 21:46 | out |
| 23 | 16:14 | out |
| 25 | 10:43 | out |
| 27 | 05:12 | out |
| 28 | 23:41 | out |
| 30 | 18:09 | out |

June

| | | |
|---|---|---|
| 1 | 12:38 | out |
| 3 | 07:07 | out |
| 5 | 01:36 | out |
| 6 | 20:05 | out |
| 8 | 14:33 | out |
| 10 | 09:02 | out |
| 12 | 03:31 | out |
| 13 | 22:00 | out |
| 15 | 16:29 | out |
| 17 | 10:57 | out |
| 19 | 05:26 | out |
| 20 | 23:55 | out |
| 22 | 18:24 | out |
| 24 | 12:53 | out |
| 26 | 07:21 | out |
| 28 | 01:50 | out |
| 29 | 20:19 | out |

July

| | | |
|---|---|---|
| 1 | 14:48 | out |
| 3 | 09:17 | out |
| 5 | 03:45 | out |
| 6 | 22:14 | out |
| 8 | 16:43 | out |
| 10 | 11:12 | out |
| 12 | 05:41 | out |
| 14 | 00:09 | out |
| 15 | 18:38 | out |
| 17 | 13:07 | out |
| 19 | 07:36 | out |
| 21 | 02:05 | out |
| 22 | 20:33 | out |
| 24 | 15:02 | out |
| 26 | 09:31 | out |
| 28 | 04:00 | out |
| 29 | 22:28 | out |
| 31 | 16:57 | out |

August

| | | |
|---|---|---|
| 2 | 11:26 | out |
| 4 | 05:55 | out |
| 6 | 00:23 | out |
| 7 | 18:52 | out |
| 9 | 13:21 | out |
| 11 | 07:50 | out |
| 13 | 02:18 | out |
| 14 | 20:47 | out |
| 16 | 15:16 | out |
| 18 | 09:45 | out |
| 20 | 04:13 | out |
| 21 | 22:42 | out |
| 23 | 17:11 | out |
| 25 | 11:40 | out |
| 27 | 06:08 | out |
| 29 | 00:37 | out |
| 30 | 19:06 | out |

September

| | | |
|---|---|---|
| 1 | 13:34 | out |
| 3 | 08:03 | out |
| 5 | 02:32 | out |
| 6 | 21:00 | out |
| 8 | 15:29 | out |
| 10 | 09:58 | out |
| 12 | 04:26 | out |
| 13 | 22:55 | out |
| 15 | 17:23 | out |
| 17 | 11:52 | out |
| 19 | 06:21 | out |
| 21 | 00:49 | out |
| 22 | 19:18 | out |
| 24 | 13:46 | out |
| 26 | 08:15 | out |
| 28 | 02:44 | out |
| 29 | 21:12 | out |

October

| | | |
|---|---|---|
| 1 | 15:41 | out |
| 3 | 10:09 | out |
| 5 | 04:38 | out |
| 6 | 23:06 | out |
| 8 | 17:35 | out |
| 10 | 12:03 | out |
| 12 | 06:32 | out |
| 14 | 01:00 | out |
| 15 | 19:29 | out |
| 17 | 13:57 | out |
| 19 | 08:26 | out |
| 21 | 02:54 | out |
| 22 | 21:23 | out |
| 24 | 15:51 | out |
| 28 | 02:39 | in |
| 29 | 21:07 | in |
| 31 | 15:35 | in |

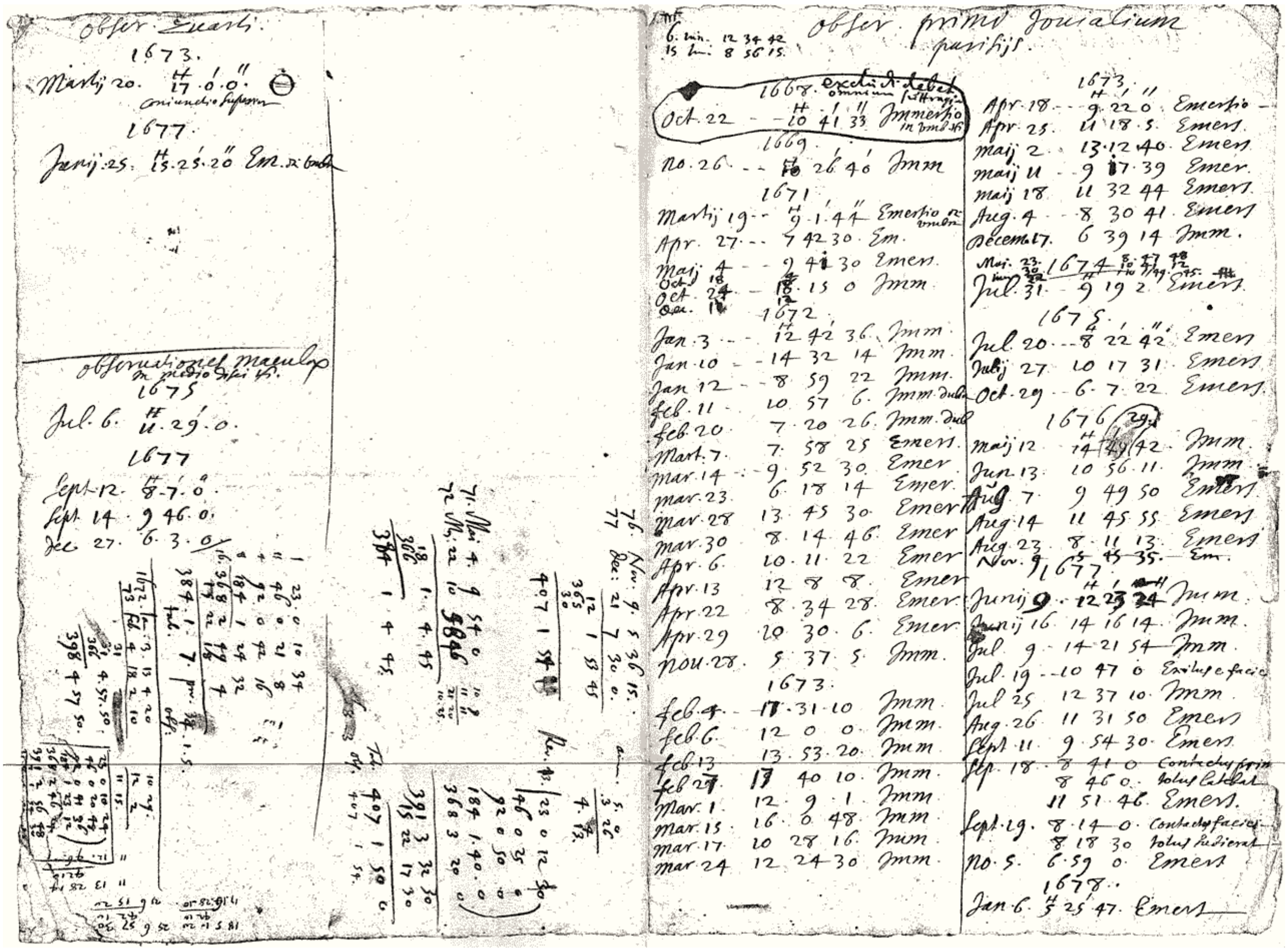
November

| | | |
|---|---|---|
| 2 | 10:04 | in |
| 4 | 04:32 | in |
| 5 | 23:01 | in |
| 7 | 17:29 | in |
| 9 | 11:58 | in |
| 11 | 06:26 | in |
| 13 | 00:54 | in |
| 14 | 19:23 | in |
| 16 | 13:51 | in |
| 18 | 08:20 | in |
| 20 | 02:48 | in |
| 21 | 21:16 | in |
| 23 | 15:45 | in |
| 25 | 10:13 | in |
| 27 | 04:41 | in |
| 28 | 23:10 | in |
| 30 | 17:38 | in |

December

| | | |
|---|---|---|
| 2 | 12:06 | in |
| 4 | 06:35 | in |
| 6 | 01:03 | in |
| 7 | 19:31 | in |
| 9 | 14:00 | in |
| 11 | 08:28 | in |
| 13 | 02:56 | in |
| 14 | 21:25 | in |
| 16 | 15:53 | in |
| 18 | 10:21 | in |
| 20 | 04:49 | in |
| 21 | 23:18 | in |
| 23 | 17:46 | in |
| 25 | 12:14 | in |
| 27 | 06:43 | in |
| 29 | 1:11 | in |
| 30 | 19:39 | in |

Such observations were made by the Danish astronomer Ole Rømer between 1668 and 1678. He used the data to demonstrate that light travels at a finite speed. (On the manuscript by Rømer shown below "Imm." represents an entry in the shadow and "Emer." indicates an exit from it.)

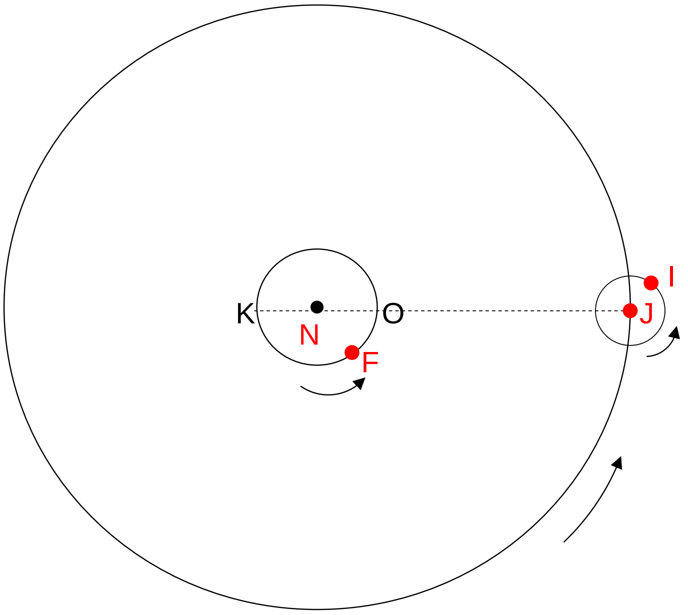


https://en.wikipedia.org

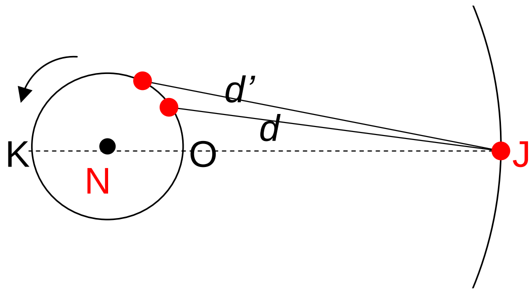
(a) Jupiter orbits around the Sun at a distance of 5.2 AU. What is the highest value of the angle NJF (Sun, Jupiter, Earth)?

(b) Io orbits close Jupiter, with an orbital radius of only six times the radius of Jupiter. Explain why one can only observe entries of Io into the shadow of Jupiter for nearly half a year – as shown in the table above – and next, for another half year, only exits are observed. (Naturally, from any given point on the Earth not all the events can be observed, since Jupiter cannot be seen during the day because of the brightness of the Sun while at other times it is covered by Earth itself.)

(c) Use the table above to determine the days of the year 2017 when the Earth was at point O and at point K, relative to Jupiter. That is, when was Jupiter in opposition (O), and in conjunction (K) relative to the Sun?




(d) Let the orbital period of Io be $T$. Assume that the Earth is receding from Jupiter (moving from O to K), and the distance of the Earth from Jupiter at the times of two subsequent exits of Io from the shadow of Jupiter are $d$ and $d' > d$, respectively. How much time elapses between two exit observations? (Take into account the time necessary for the propagation of light.)
When the distance at the subsequent exit equals $d''$, how much time elapses between the observation of the first and this third exit?




(e) If the exit after opposition is considered the zeroth exit, read the times of the zeroth, fifteenth and thirtieth exits from the table.

(f) At the time of opposition the distance between the Earth and Jupiter was $d_0$ = 4.46 AU. Determine the distance of Earth and Jupiter at the times of the zeroth, fifteenth and thirtieth exits.

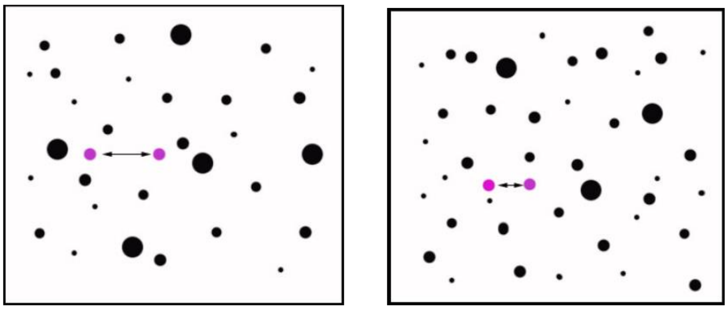
(g) Based on the results, what is the speed of light?

# 2. Old measurements using modern data and technology

## THE PARALLAX OF STARS: BESSEL'S MEASUREMENT

**2.5** The concept of parallax in general means the experience of looking at a close object from a somewhat different angle, and observing that it seems to be shifting against the background of distant objects. The annual parallax of stars is thus a fluctuation of the apparent position of the stars due to the orbiting of the Earth around the Sun. The figures below represent two observations of each of two stars nearby, Almahab, and Bechamel. The time elapsed between two distinct observations of the same star was 6 months.
(a) Compare the distances to the two stars.

Almahab Bechamel

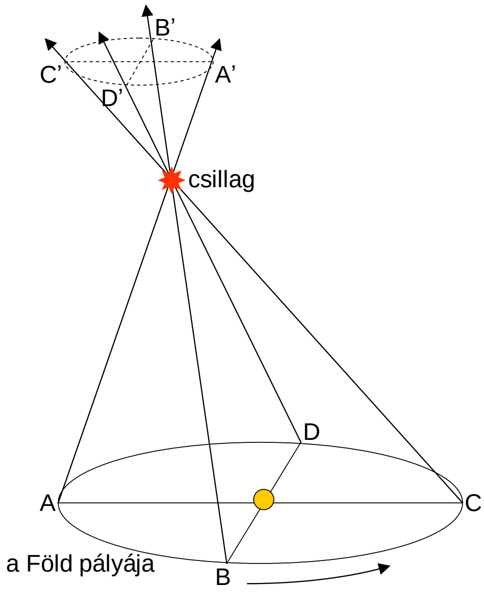


(b) The phenomenon of parallax results in an apparent displacement of the star closest to the Sun, α Centauri by 1.49 arc seconds on the sky, while the apparent displacement of the brightest star of the sky, Sirius caused by parallax is 0.753 arc seconds. What are the distances to these stars in many light years?

**2.6** Tycho Brahe and other astronomers in the 16th century made an attempt to demonstrate the annual parallax of stars by observations made with the naked eye, in order to decide upon the disputed issue of the Earth orbiting the Sun which was gaining ground at the time. No parallax was observed by them.
The resolving power of the human eye is approximately 0.02 degrees. The closest star is about 4.3 light years away (this is α Centauri, further south than –60°, therefore Tycho Brahe could not see it).
(a) What is the maximum possible angular displacement that may occur between two observations of α Centauri?
(b) By what factor is α Centauri more distant than the distance from which the parallax could be seen with the naked eye?

**2.7** For a long time, the lack of parallax observations, that is, no annual changes seen in the apparent direction of stars, remained an annoying counter-argument against the heliocentric system,. According to expectations, in order to observe the star on the figure next page the telescope needs to be directed to point A' in the point A of the Earth orbit, while C' would be the correct direction in the opposite point C.
(a) When the advancement of technology allowed, it was demonstrated that indeed, all stars exhibit an annual fluctuation of approximately 40-seconds. If this was the desired annual parallax, how far these stars would be?

(b) Now we are aware that stars are a lot more distant than that, and are not all at the same distance, but at the beginning of the 18th century such a result could have been believable. However, something was wrong. The fluctuation between directions A' and C' was not observed between the points A and C of the Earth's orbit: Instead, the telescope had to be aimed at A' when the Earth was at point B, and in direction C' when the Earth was at point D. In the same way, the instrument had to be turned towards D' at point A and towards direction B' at position C. In other words, the shift showed a regular delay of a quarter year compared to the expected positions.

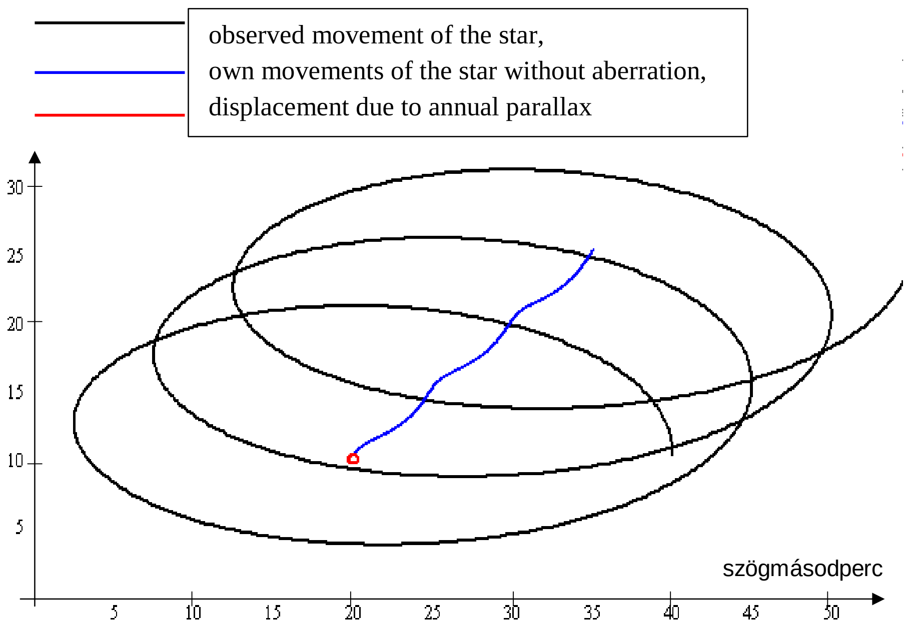



star, Earth orbit

The explanation was given by James Bradley in 1729: the fluctuation observed is caused by the fact that light needs time to travel down the telescope and during this time the Earth progresses somewhat on its orbit.
What is the angle you have to tilt your telescope relative to the real direction of the star if you are to study a star which is located in a direction exactly perpendicular to the plane of the Earth's orbit?

(c) What annual fluctuation does this cause in the direction of the star?

**2.8** Though the fact of Earth orbiting the Sun was proved by the discovery of the aberration of light (see the previous exercise) at the end of the 18$^{th}$ century, the annual parallax of stars resulting from the orbiting motion could not be demonstrated for a long time still (consequently, it could was not known how far in reality stars are). Stellar parallax was finally successfully demonstrated by Friedrich Bessel in 1838. He chose the star named 61 Cygni in the constellation of Cygnus, since 61 Cygni seems to be displaced relatively quickly, with a speed of 5 angular seconds each year relative to the other stars around, therefore it can be assumed to be closer to us and hence, its parallax can be observed with a greater chance.




angular seconds

http://astronomie-smartsmur.over-blog.com

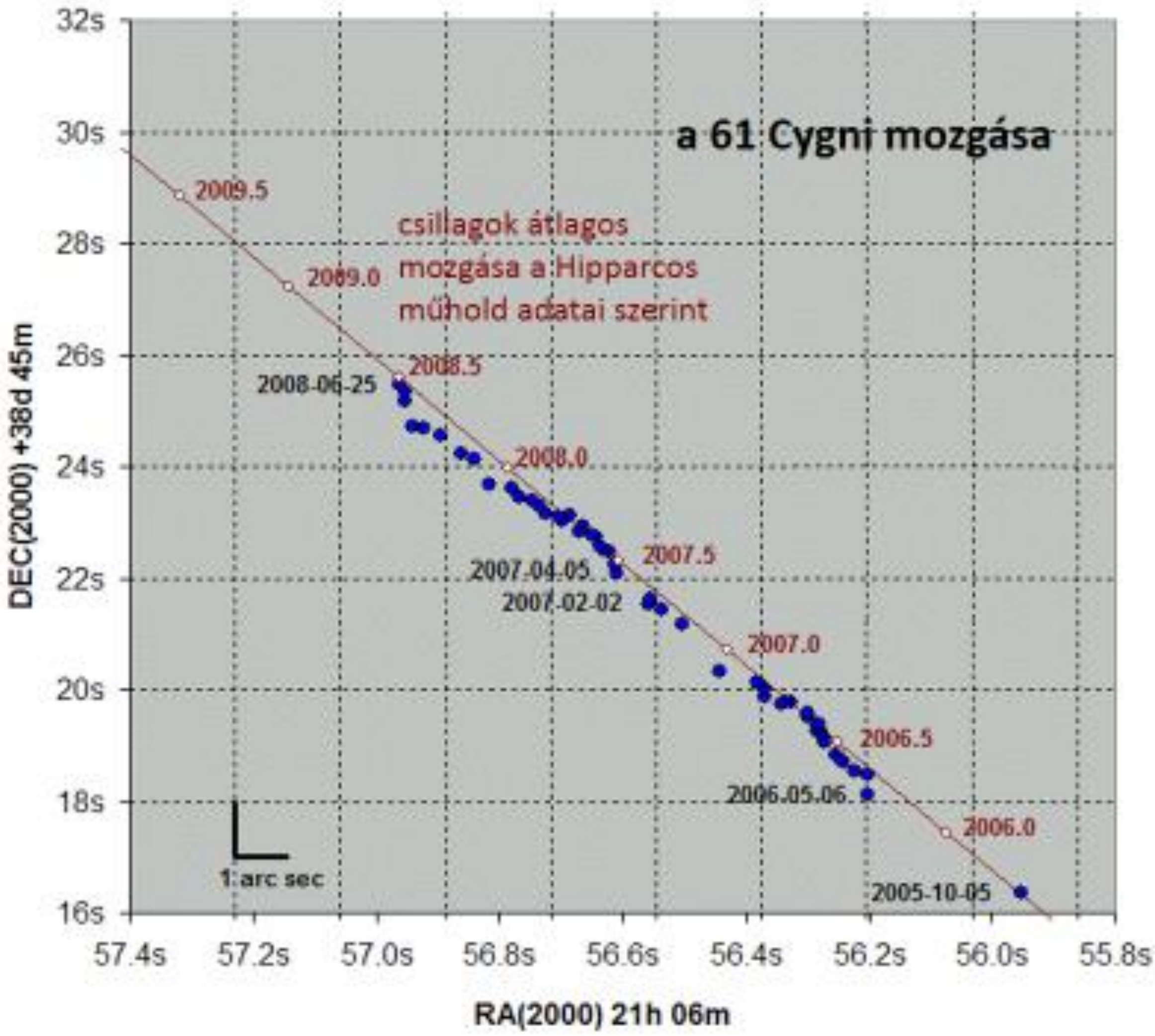
(c) 2005. és 2009. között Bessel távcsövéhez nagyon hasonló műszerrel megismételték Bessel megfigyeléseit. Az ábra már modern eszközökkel készült. A következő ábrán az egyenes olyan csillagok átlagos helyzetét ábrázolja, amelyek feltehetőleg az aberráción kívül más elmozdulást nem mutatnak.
Due to the proper motion of the star the parallax was expected to appear as an annual cyclic undulation superposed onto the proper motion, as shown by the blue curve on the figure. However, the phenomenon of aberration causes twice as big fluctuations as the parallax, therefore the apparent path of the star on the sky is the black curve, in which the annual ripples go almost unnoticed.

(a) Why does the black spiral only show the 40’’ annual aberration (calculated in the previous exercise) in a single direction (horizontally), while deviations in the other direction are a lot smaller?

(b) What can be done to obtain the blue curve instead of the observed black curve?

(c) Bessel’s observations were repeated between 2005 and 2009 using instruments very similar to his telescope. The figure was drawn with modern technology. The line on the following figure shows the mean positions of stars which were assumed to exhibit no displacement other than aberration.

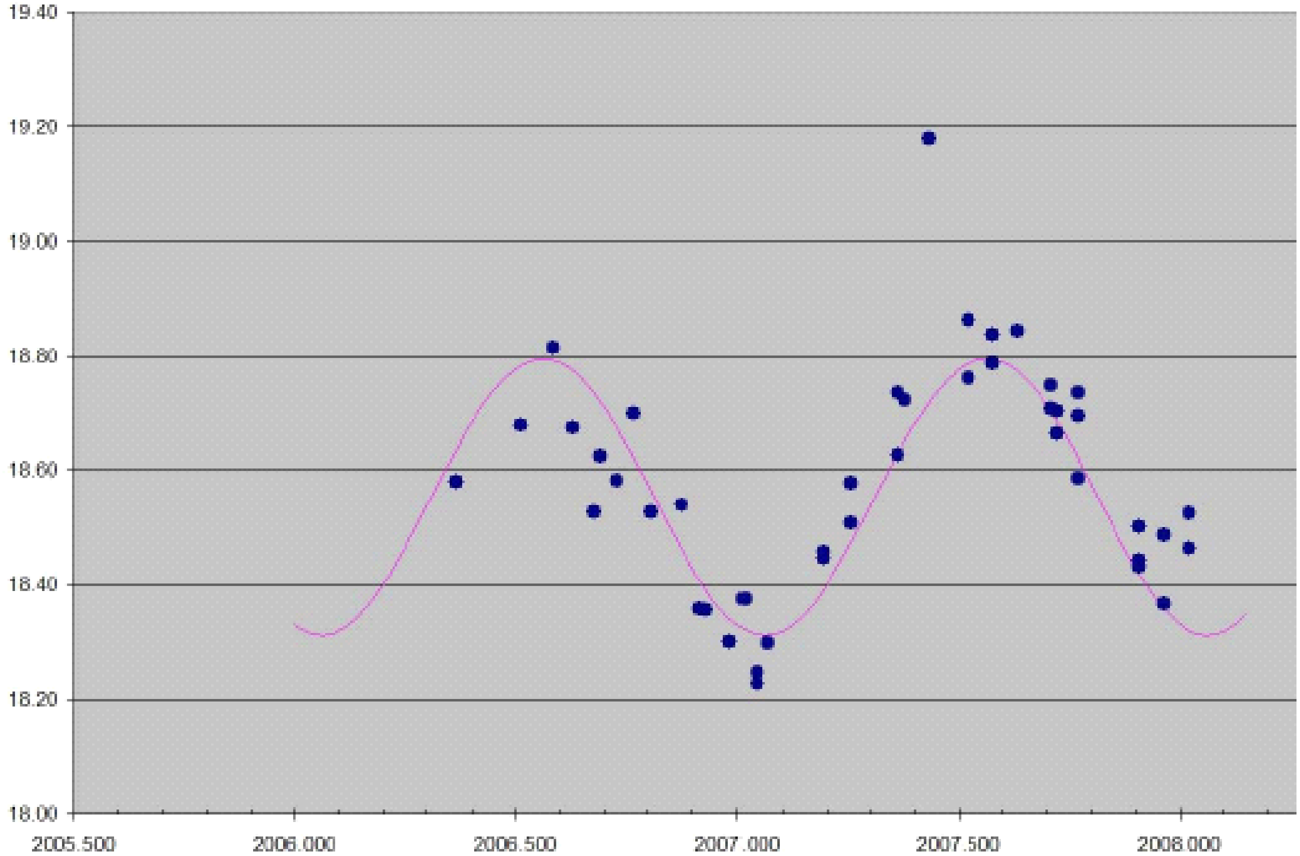
average motion of starts according to the data of the satellite Hipparcos, motion of 61 Cygni




http://www.jdso.org/volume4/number2/Vollmann_74_77.pdf
http://www.richweb.f9.co.uk/astro/nearby_stars.htm

By subtracting the values corresponding to the line from the waving set of data, the desired annual fluctuation is received. The figure overleaf shows the position measured in arc seconds as a function of time measured in years.

Based on the measurement data, what is the distance to 61 Cygni?




http://www.jdso.org/volume4/number2/Vollmann_74_77.pdf

# Solution 2.

**2.1** (a) Long shadows providing a good contrast are seen at the quarter phases. (At full Moon tthe Sun is two high above the surface of the Moon, and filters may be needed to reduce brightness.) At last quarter the Moon rises only at midnight, therefore first quarter is best.

(b) $\frac{1}{20} = \frac{DC}{2r}$,

$DC = 0.1r$

$$r^2 + (0{,}1r)^2 = (r+h)^2$$

$$r^2 + 0{,}01r^2 = r^2 + 2rh + h^2$$

$h$ is small next to the radius, $h^2$ can be neglected:

$$0{,}01r^2 = 2rh$$

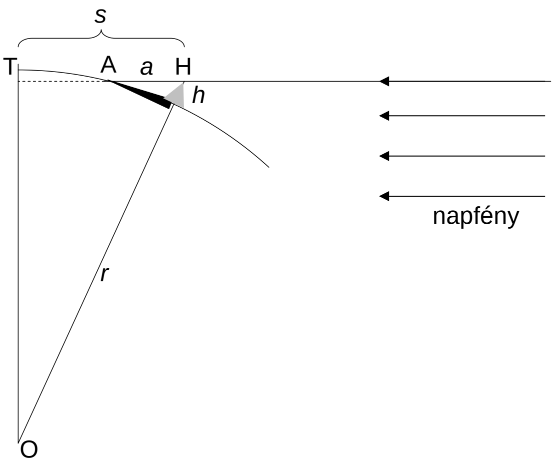
$$0{,}01r = 2h$$

$$h = \frac{r}{200} \approx 9000\text{m}.$$

**2.2** The figure shows the mountain and the shadow a bit exaggerated. At first or last quarter the landscape is seen from above, length of shadow is $a$, distance from terminator is $s$.
The mountain is low compared to the radius, so similar triangles can be used:

$$\frac{h}{a} = \frac{s}{r}$$

$$h = \frac{as}{r}$$




Determination of the scale factor:

If the length of the chord is 13.0 cm, and the height of the segment is 2.7 cm, then

$$r^2 = 6{,}5^2 + (r - 2{,}7)^2$$

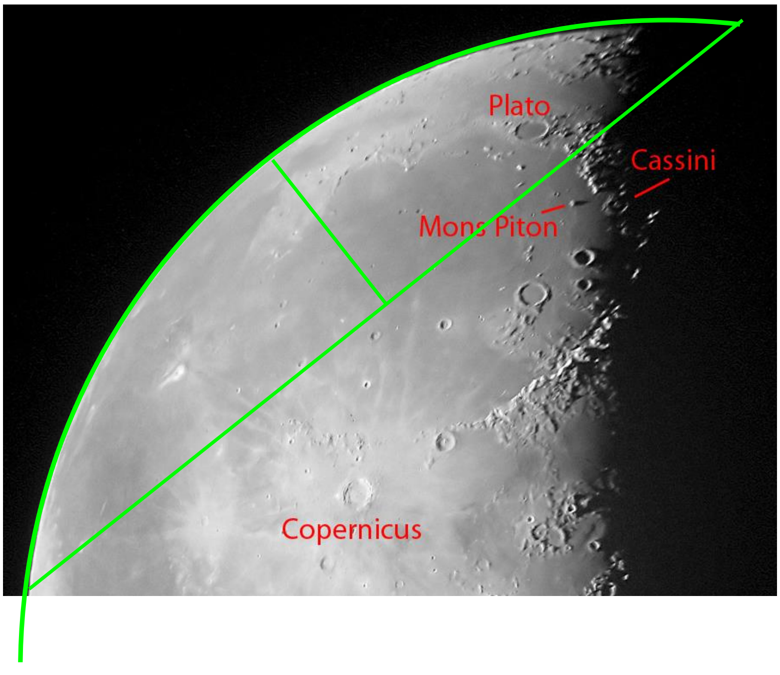
$r$ = 9.2 cm,
corresponding to 1740 km.

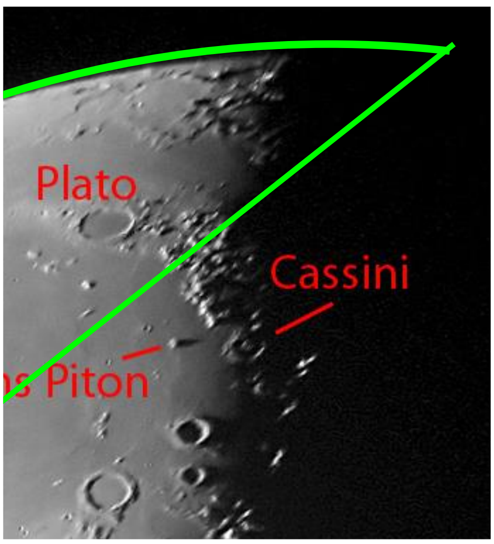
In order to measure $a$ and $s$ it helps to magnify the picture.
In a threefold magnification 27.5 cm corresponds to 1740 km.

The length of the shadow is $a \approx 0.7$ cm, distance of the peak from the terminator is $s \approx 1.7$ cm (it is difficult to locate the terminator). Using these values

$$h = \frac{as}{r} = \frac{0{,}7 \cdot 1{,}7}{27{,}5} = 0{,}043\text{cm},$$

meaning a mountain approximately 3 km high.



**2.3** (a) If the diameter of the circle is 15 cm (corresponding to a distance of 2 AU= $3\cdot10^{11}$ m) and the pencil line is about 0.05 mm, then its thickness is

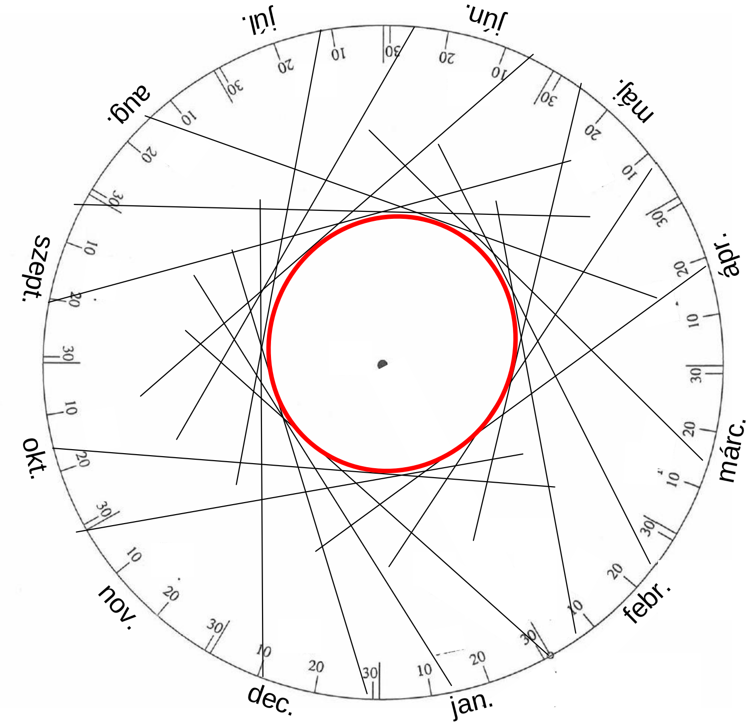
$$5\cdot10^{-5}\cdot\frac{3\cdot10^{11}}{0{,}15}=1\cdot10^{8}\,\mathrm{m}=100000\mathrm{km}$$

(c) Based on the figure below $a$ = 0.38 AU (figure from literature 0.39 AU)

(d) $0.38^{3/2}$ = 0.23 years = 86 days (figure from literature 88 days)

(e) Eccentricity based on the figure above is 0.17 (figure from literature 0.21)

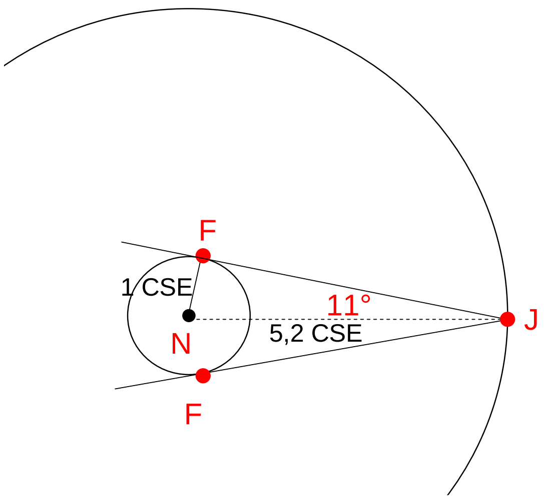
(b)



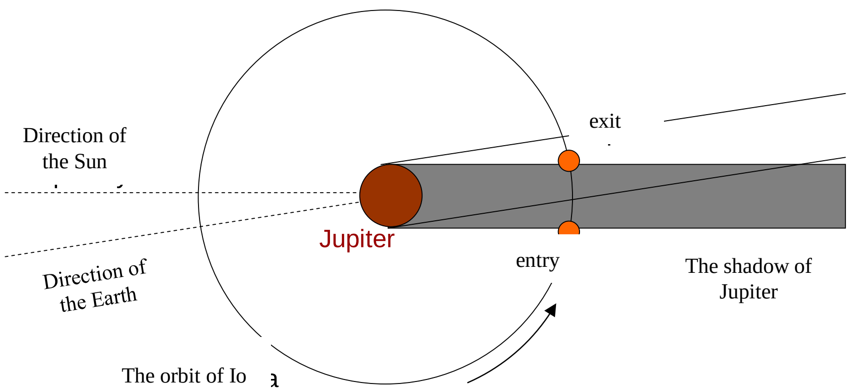

**2.4** (a) The angle is greatest when JF touches the Earth's orbit (Jupiter is in quadrature with the Sun). In the right triangle NFJ

$$\sin\alpha=\frac{1}{5{,}2}\cdot$$

$\alpha = 11^{\circ}$




(b) For simplicity, assume that the three objects are in the same plane (this is roughly so), and disregard Jupiter's motions.
As the Earth moves from K to O only the exit of Io is seen since Jupiter covers it up upon entry.



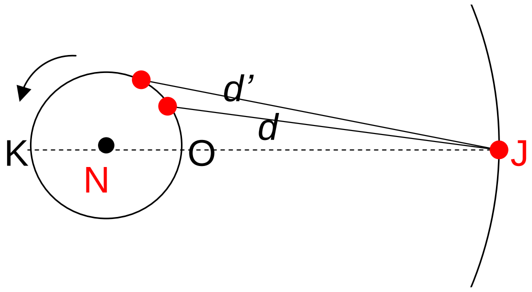
(c) Opposition: 7 April, conjunction: 26 October

(d) If one exits takes place at time $t$, the next one will be at $t + T$.
If the speed of the light were infinite, a time of exactly $T$ time would pass between observations. However since the speed of light is $c$, the times of the two observations will be

$$t+\frac{d}{c}, \text{ and } t+T+\frac{d'}{c}, \text{ respectively,}$$

thus the time in between them would be

$$T+\frac{d'-d}{c}.$$




$$T+\frac{d'-d}{c}+T+\frac{d''-d'}{c}=2T+\frac{d''-d}{c}$$

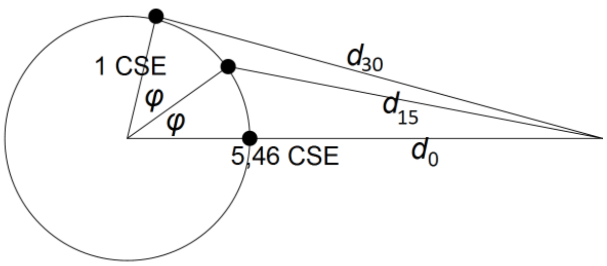
will pass between observations one and three, thus the total of delays will be the full increase of distance divided by the speed of light.

(e) $t_0$ = 7 April 15:49,
$t_{15}$ = 4 May 04:58,
$t_{30}$ = 30 May 18:09,

(f) $d_0$ = 4.46 AU

$d_{15}$ and $d_{30}$ can be determined using the cosine rule:

$t_{15} - t_0$ = 26 days 13 hours 9 minutes,
$t_{30} - t_{15}$ = 26 days, 13 hours 11 minutes,
thus each is approximately 26.55 days.




During 26.55 days the angular displacement of the Earth on its orbit is

$$\varphi = 360°\cdot\frac{26{,}55}{365} = 26{,}2° .$$

$$d_{15}^2 = 1^2 + 5{,}46^2 - 2\cdot 1\cdot 5{,}46\cdot\cos 26{,}2°$$

$d_{15}$ = 4.58 AU

$$d_{30}^2 = 1^2 + 5{,}46^2 - 2\cdot 1\cdot 5{,}46\cdot\cos(2\cdot 26{,}2°)$$

$d_{30}$ = 4.91 AU

(g)
$$26\text{ nap }13\text{ óra }9\text{ perc} = 15T + \frac{d_{15}-d_0}{c}$$
$$26\text{ nap }13\text{ óra }11\text{ perc} = 15T + \frac{d_{30}-d_{15}}{c}.$$

A különbség

$$2\text{ perc} = \frac{d_{30}+d_0-2d_{15}}{c} = $$
$$= \frac{4{,}91+4{,}46-2\cdot 4{,}58}{c} = \frac{0{,}21}{c}$$

vagyis a fény 2 perc alatt 0,21 CSE távolságot tesz meg, tehát

$$c = \frac{0{,}21\cdot 1{,}5\cdot 10^{11}}{120} = 2{,}6\cdot 10^8\,\text{m/s}$$

(g)
$$26\text{ nap }13\text{ óra }9\text{ perc} = 15T + \frac{d_{15}-d_0}{c}$$
$$26\text{ nap }13\text{ óra }11\text{ perc} = 15T + \frac{d_{30}-d_{15}}{c}.$$

The difference is

$$2\text{ perc} = \frac{d_{30}+d_0-2d_{15}}{c} = $$
$$= \frac{4{,}91+4{,}46-2\cdot 4{,}58}{c} = \frac{0{,}21}{c}$$

in other words the light covers a distance of 0.21 AU in two minutes, therefore

$$c = \frac{0{,}21\cdot 1{,}5\cdot 10^{11}}{120} = 2{,}6\cdot 10^8\,\text{m/s}$$

**2.5** (a) Since Almahab has a parallax twice of that of Bechamel, Bechamel must be twice as far away as the Almahab.

(b) If the distance of the star is $d$, the diameter of the Earth orbit is $D$, the angle of vision of the Earth orbit from there (expressed in radian) is $d/D$.

1.494” = $7.25\cdot 10^{-6}$ rad and
0.753” = $3.65\cdot 10^{-6}$ rad.

The distance of α Centauri will be

$$d = \frac{3{,}0\cdot 10^{11}}{7{,}25\cdot 10^{-6}} = 4{,}14\cdot 10^{16}\,\text{m} = 4{,}4\text{ fényév},$$

That of Sirius

$$d = \frac{3{,}0\cdot 10^{11}}{3{,}65\cdot 10^{-6}} = 8{,}22\cdot 10^{16}\,\text{m} = 8{,}7\text{ fényév}.$$

That is, Sirius is about twice as distance and accordingly, its apparent displacement is half that much (just like in the case of the stars in exercise (a)).

**2.6** (a) The α Centauri is $4.1 \cdot 10^{16}$ m away. The angle of view of the Earth orbit from this distance is

$$\frac{2 \cdot 1{,}5 \cdot 10^{11}}{4{,}1 \cdot 10^{16}} = 7{,}3 \cdot 10^{-6}\,\text{rad} = 1{,}5''.$$

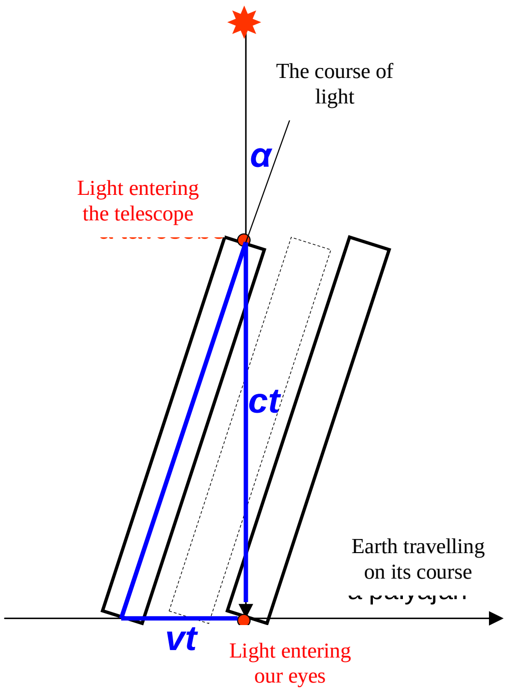
(b) 0.02° = 72’’, this is a displacement about 50 times larger, α Centauri is approximately 50 times farther.

**2.7** (a) 40’’ = $1.9 \cdot 10^{-4}$ rad

The diameter of the Earth orbit is seen in a 40’’ angle:

$$d = \frac{2 \cdot 1{,}5 \cdot 10^{11}}{1{,}9 \cdot 10^{-4}} = 1{,}5 \cdot 10^{15}\,\text{m} = 0{,}16\,\text{fényév}$$

(b) If Earth travels at a speed of $v$ on its orbit, and light passes on the length of the telescope in time $t$, Earth will cover a path $vt$.




Direct the telescope so that its top be above our head $t$ time earlier than the moment the light enters our eyes.

Since Earth orbits around the Sun with a speed of 30 km/s, and the speed of light (known to Bradley) is 300 000 km/s,

$$\tan\alpha = \frac{vt}{ct} = \frac{v}{c} = \frac{30}{300000} = 1{,}0 \cdot 10^{-4}$$

$\alpha$ = 20.6 angular seconds

(c) $2\alpha = 41$ angular seconds, in line with the value around 40’’ observed.

Note:

1. Even though the annual parallax of the stars could not be observed up to date, this ±20 seconds fluctuation known as aberration provided finally an experimental proof to the orbiting of Earth nearly 200 years after Copernicus, and could not be explained by the geocentric model.

2. Since the fluctuation is superimposed on all other movements observed, you need to correct it whenever there is a need for higher accuracy.

**2.8** (a) The 61 Cygni is not situated perpendicular to the plane of the Earth orbit, the maximum 40’’ oscillation is seen only when the Earth travels perpendicular to the direction of the star. From a different angle the perpendicular component of the speed is less.

(b) The aberration is the same for both 61 Cygni, and the other stars nearby. If the position of 61 Cygni is studied in relation to another star “beside it”, you will get the blue curve.

(c) The sinus curve fit on the graph varies between approximately 18.30 and 18.80 angular seconds, thus the amplitude is 0.25’’.
The 61 Cygni is

$$d = \frac{1}{p} = 4{,}0\text{pc} = 13\text{fényév}$$ light years away

Note:
Bessel obtained 0.31 angular seconds, the value derived from the most modern technology is 0.287’’. His result can be seen as brilliant, since the diameter of this telescope was merely 158 mm, thus the stars appeared in the field of vision as discs with a diameter or almost 1’’.

# 3. Examination of functional dependencies
## LINEAR APPROACH

**3.1** It is known that the rotational period of the Earth is 24 hours, or more precisely 23 hours, 56 minutes and 4 seconds. It was not always as much: the next table shows how many days you had in a year in the different geological ages based on the examination of the marine deposits and sediments, that is, how many times Earth turned around itself during a cycle period around the Sun. (Assuming that the Sun orbiting period was not subject to change in the meantime.)

| **Geological ages** | **Million years ago** | **Number of days in a year** | **Length of days (hour)** | **Difference (hours)** |
|---|---|---|---|---|
| Holocene | 0 | 365 | | |
| Cretaceous | 70 | 370 | | |
| Triassic | 220 | 372 | | |
| Permian | 290 | 383 | | |
| Early Carbonaceous | 340 | 398 | | |
| Late Devonian | 380 | 399 | | |
| Middle Devonian | 395 | 405 | | |
| Early Devonian | 410 | 410 | | |
| Late Silurian | 420 | 400 | | |
| Middle Silurian | 430 | 413 | | |
| Early Silurian | 440 | 421 | | |
| Late Ordovician | 450 | 414 | | |
| Middle Cambrian | 510 | 424 | | |
| Neoproterozoic | 600 | 417 | | |
| Mesoproterozoic | 900 | 486 | | |

http://spacemath.gsfc.nasa.gov

(a) Compute how many hours days were in each of the ages shorter than today.

(b) Illustrate the data as a function of the time elapsed, fit a line on the points, and determine how many seconds days have become longer during a century in average.

(c) Due to the friction of the ebb and flow, in the distant future both the moon month and the earthly days will last for a period of 47 present days. How much will be the distance between the Earth and the Moon at this time?

(d) Could we use the finding obtained in exercise (b) to determine the time when it will more or less occur?

**3.2** The interplanetary space is not quite empty, there are a number of things in it: pieces of asteroids, comets, leftover material from the creation of planets, etc. The surface of our own planet is also showered by pieces of rock and debris at speed of up to several ten km/s. The figure below shows the number of object of this scale which hit a 1 $km^2$ area. The scale on both axises of the figure is logarithmic, since both the mass and the number varies in several orders of magnitude.

(a) The diameter of a round meteoroid with a density of 3 $g/cm^3$ is 4 cm. How much is its mass?

(b) How many of such objects hit the Earth in a 10 000 $km^2$ area?

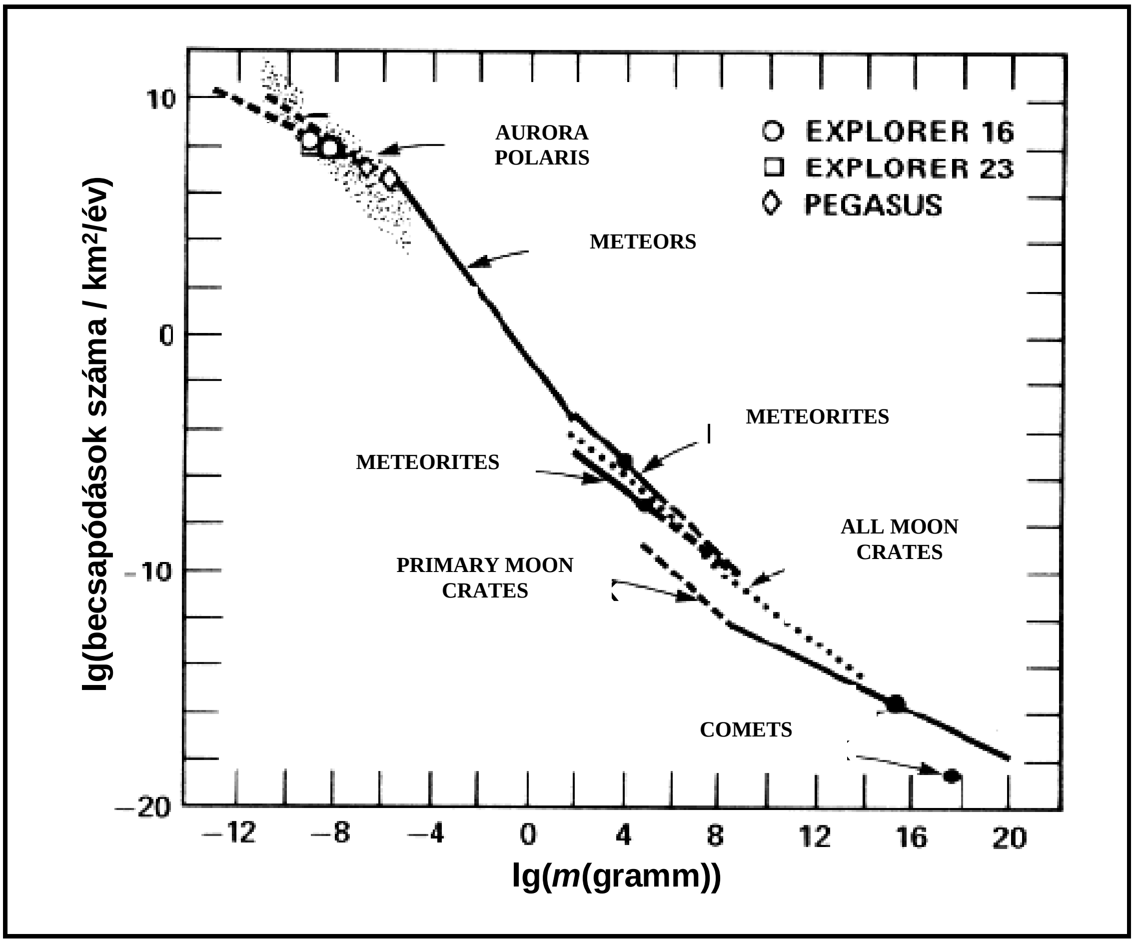
(c) Approximate the range of the graph form 1 gram to $10^{20}$ grams using a straight line.

*The next question leads to an integration exercise.*

(d) How many meteorites collide the Earth in the d*m* mass interval around *m*?

(e) How many tons of matter hits the full surface of the Earth annually in the 1- $10^{20}$ grams range?

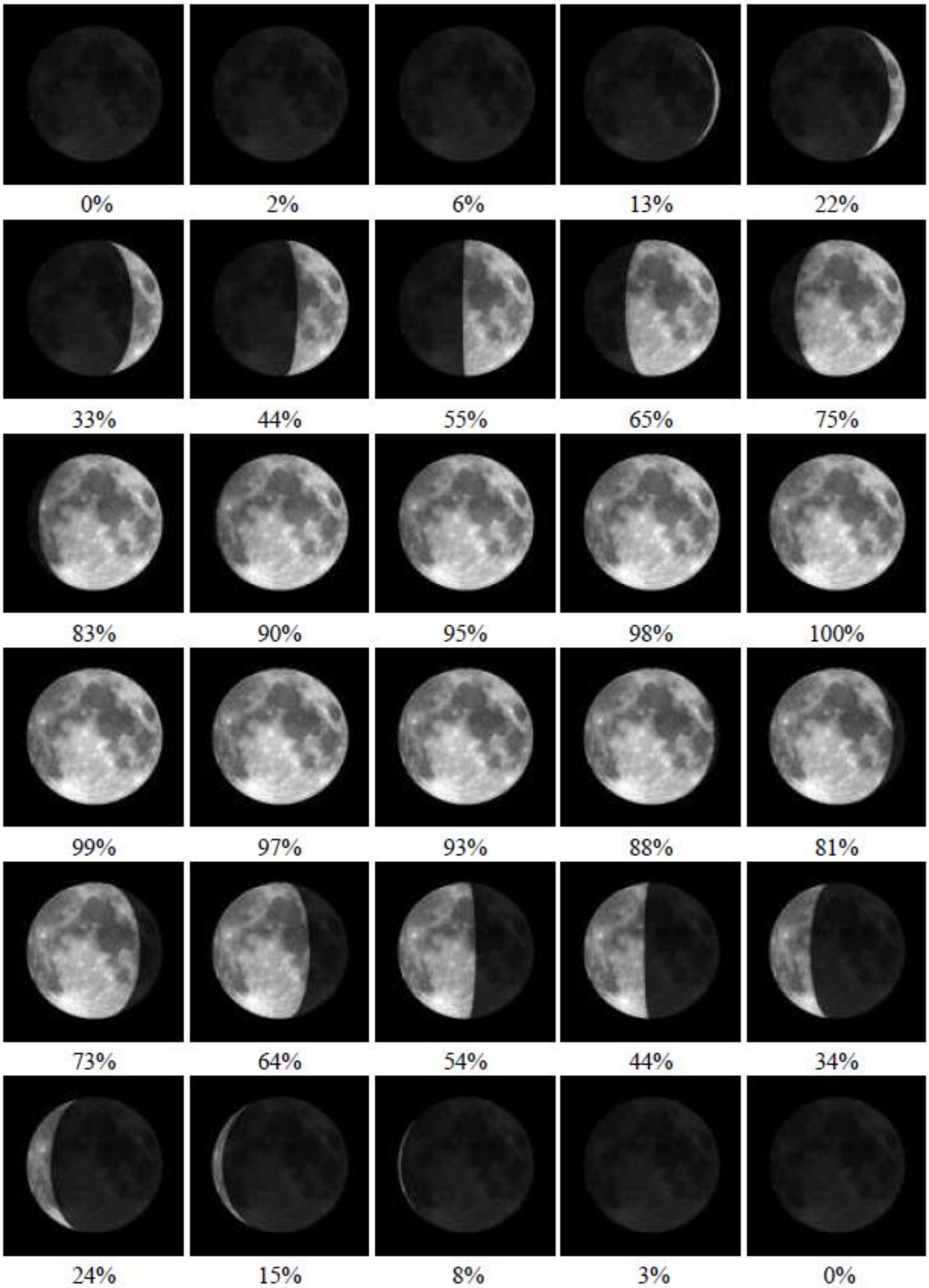
Number of impacts km$^2$/year




http://spacemath.gsfc.nasa.gov

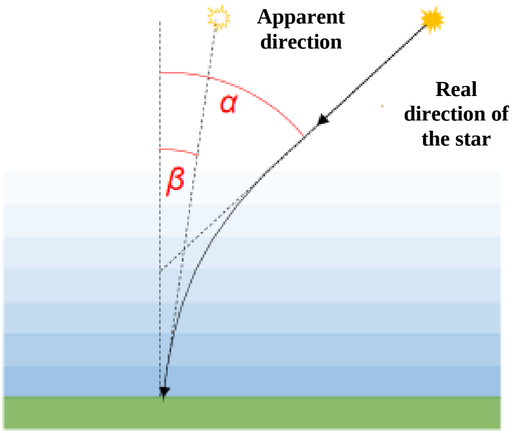
# 3. Examination of functional dependencies
## NON LINEAR CORRELATIONS

**3.3** These pictures taken across a moon month show the percentage of the disc of the Moon illuminated. Represent the area lit as a function of time.
What kind of a function did you get? Explain the results.



**3.4** The optical refractive coefficients of the various layers in the atmosphere are different, therefore the direction of the light arriving to the telescope differs from the real direction of the celestial body observed.

(a) If the refractive coefficient of air at ground level is 1.000292, and a star is observed in an angular distance of 13°37'52'' from the zenith, how distant it is from the zenith in reality?




(b) The calculation of the refractive coefficient of air with a temperature of $t$ °C and pressure $p$ bar can be accomplished using the following correlation:

$$n = 1 + 0.000292 \cdot \frac{p}{1{,}014} \cdot \frac{273}{273 + t} .$$

How much the apparent position of the star above would change and in which direction if temperature rises from 0°C to 15°C, while pressure drops from 1014 hPa to 1010 hPa?

(c) Represent the apparent shift of the zenith distance $\delta = \alpha - \beta$ obtained from the drawing above as a function of the real zenith distance $\alpha$.

(d) The correlation obtained on the basis of the drawing above can only be used for low angles in practice (at the initial section of the graph, close to linear). For large angles, in other words when the object examined is close to the horizon) the difference grows more and more rapidly indeed, this is why you see the Sun or the Moon "flat" when close to the horizon, but the formula describing the relationship is more complicated than that above.

Why the correlation obtained from the drawing above provides incorrect results for large zenith distances?

http://www.atoptics.co.uk/atoptics

# Solution 3.

**3.1** (a)

| **time passed (million years)** | **number of days in a year** | **length of days (hour)** | **difference (hour)** |
|---|---|---|---|
| 0 | 365 | 24.0 | 0.0 |
| 70 | 370 | 23.7 | 0.3 |
| 220 | 372 | 23.5 | 0.5 |
| 290 | 383 | 22.9 | 1.1 |
| 340 | 398 | 22.0 | 2.0 |
| 380 | 399 | 22.0 | 2.0 |
| 395 | 405 | 21.6 | 2.4 |
| 410 | 410 | 21.4 | 2.6 |
| 420 | 400 | 21.9 | 2.1 |
| 430 | 413 | 21.2 | 2.8 |
| 440 | 421 | 20.8 | 3.2 |
| 450 | 414 | 21.2 | 2.8 |
| 510 | 424 | 20.7 | 3.3 |
| 600 | 417 | 21.0 | 3.0 |
| 900 | 486 | 18.0 | 6.0 |

(b) The gradient of the graph is 0.006 hour / million years

$= 6\cdot 10^{-7}$ hours / 100 years = 2 ms/century

(c) The Earth mass is 81 times as much as that of the Moon. The centre of mass of the system divides distance in inverse proportion with masses. If the distance looked for is 82*d*, masses are *M* and *M*/81, the orbital radiuses are *d*, and 81*d*, respectively.

$$\frac{\gamma M\cdot(M/81)}{(82d)^2}=M\cdot d\cdot\frac{4\pi^2}{T^2}$$

$$\frac{\gamma(M/81)}{82^2\cdot d^2}=d\,\frac{4\pi^2}{T^2}$$

$$d^3=\frac{\gamma(M/81)\cdot T^2}{82^2\cdot 4\pi^2}$$

$$82d=\sqrt[3]{\frac{82\cdot 6{,}67\cdot 10^{-11}\cdot 6{,}0\cdot 10^{24}\cdot(47\cdot 24\cdot 3600)^2}{81\cdot 4\pi^2}}$$

$$82d=5{,}5\cdot 10^8\,\mathrm{m}$$

(in other words, somewhat less than one and a half of the current distance).

Or: using Kepler's third law

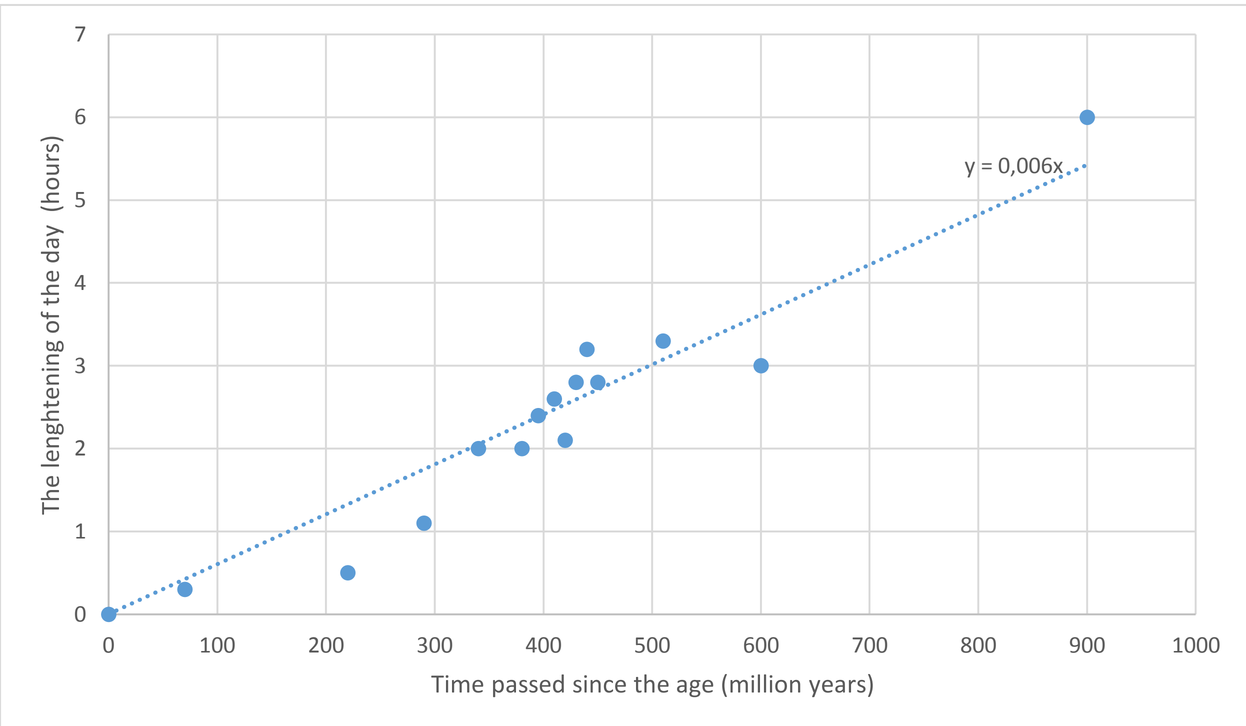
$$\frac{a^3}{T^2}=\frac{\gamma(M+M/81)}{4\pi^2}$$

$$a^3=\frac{82\gamma MT^2}{81\cdot 4\pi^2}$$

$$a=\sqrt[3]{\frac{82\cdot 6{,}67\cdot 10^{-11}\cdot 6{,}0\cdot 10^{24}\cdot(47\cdot 24\cdot 3600)^2}{81\cdot 4\pi^2}}$$

$$a=5{,}5\cdot 10^8\,\mathrm{m}.$$

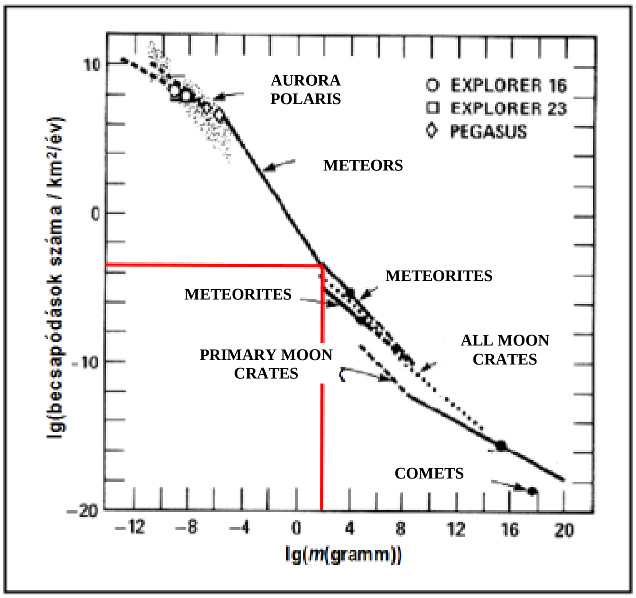
(d) No. Albeit the time series was approached using a linear function for the sake of simplicity, there is no reason to believe that such a correlation would be linear indeed. The 47 hours day should be imagined that the length of the days will getting closer to this limit value.



**3.2.** (a) $m = \rho \cdot V = \frac{4}{3}\rho\pi \cdot r^3 =$

$$= \frac{4}{3} \cdot 3000 \cdot \pi \cdot 0{,}02^3 = 0{,}1\text{kg} = 1 \cdot 10^2\,\text{g}$$

(b) Reading from the graph the frequency is approximately $10^{-3.5} \approx 0.00032$ per $km^2$ per year, in other words about three impacts in every 10 000 $km^2$.




(c) Using a rough linear approach the angular coefficient (gradient) is –0.8 and the axial intersection is at approximately –3.

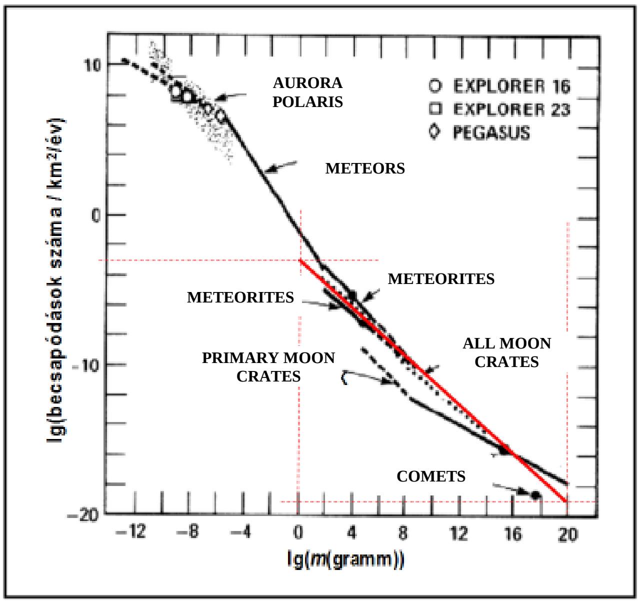
The formula of the graph is

$$\lg N = -0{,}8 \lg m - 3$$

(d) Annual meteorite impacts in the d*m* mass interval around *m*:

$$N(m) = 0{,}001 \cdot m^{-0.8}\text{d}m\,.$$

**3.3**




(e) Total impacting mass on a unit of surface:

$$M = \int_1^{10^{20}} 0{,}001 \cdot m^{-0{,}8}\text{d}m = \left[0{,}001 \cdot \frac{1}{0{,}2} m^{0{,}2}\right]_1^{10^{20}} =$$

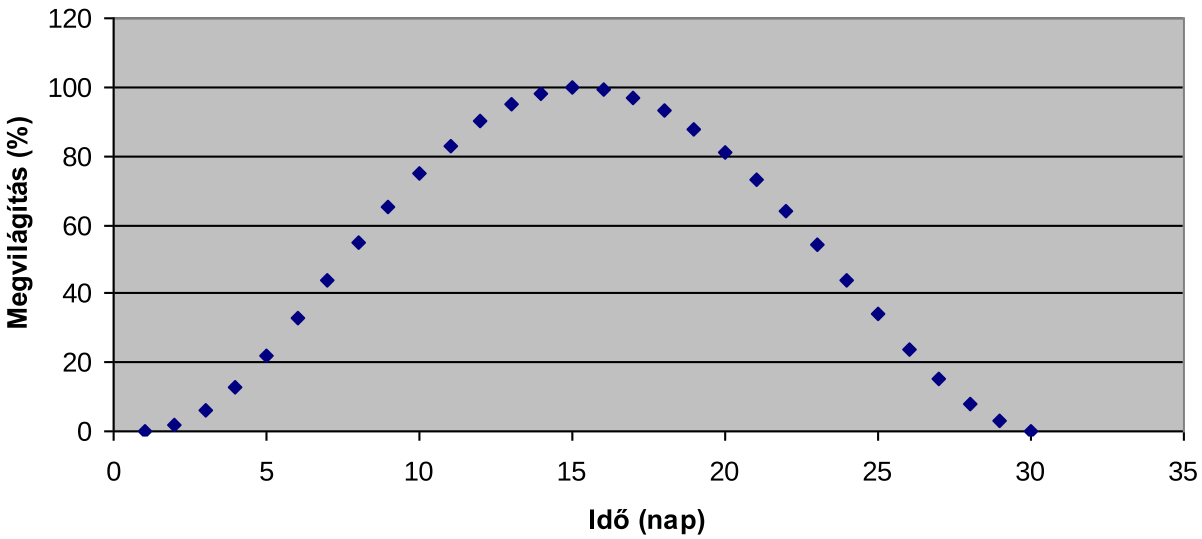
$$= 0{,}005\left(\left(10^{20}\right)^{0{,}2} - 1^{0{,}2}\right) =$$

$$= 0{,}005(10000 - 1) \approx 50\text{g}/\text{km}^2/\text{év}$$

The surface area of the Earth is $4\pi R^2 = 4\pi \cdot 6400^2 \approx 5 \cdot 10^8\,\text{km}^2$.
Thus the total mass of impacting meteorites

$$5 \cdot 10^8\,\text{km}^2 \cdot 50\text{g} = 3 \cdot 10^{10}\,\text{g} = 30000\text{tonna}$$

Note:
The estimate is very sensitive to the matches line and the boundaries of integration. Possible figures vary between 20 000 and 100 000 tons.

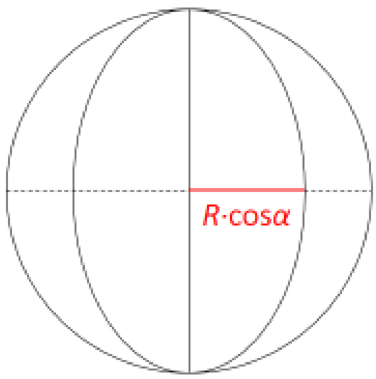



Illumination Time (days)

You get a sinus quadrate graph.

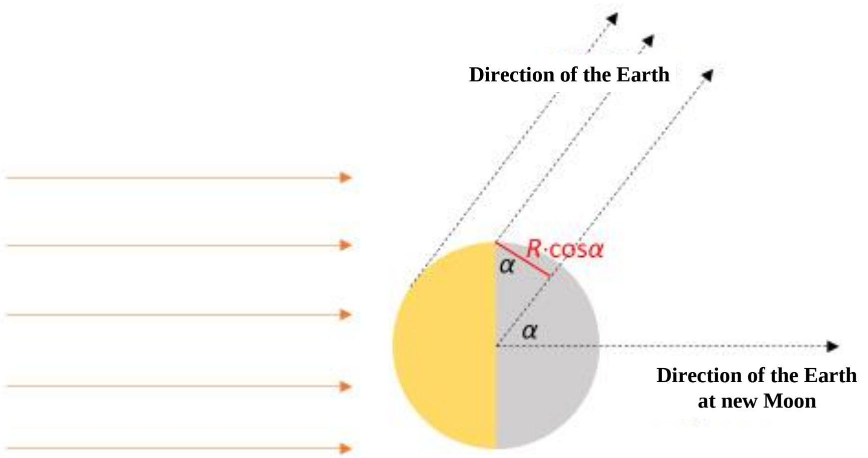
Explanation:

If the angular rotation of the orbiting Moon calculated from new Moon seen from the Earth is $\alpha$, the area of the crescent can be derived from the half moon less the half ellipse obtained by $\cos\alpha$ compression ratio (meaning an increase in the surface when $\alpha$ is a blunt angle):




$$\frac{R^2\pi}{2} - \frac{R \cdot R\cos\alpha \cdot \pi}{2} =$$

$$R^2\pi \cdot \frac{1-\cos\alpha}{2} = R^2\pi \cdot \sin^2\frac{\alpha}{2}$$




**3.4** (a) If $n_0$ is the refractive coefficient at the lower atmosphere, and the atmosphere is imagined as an $N$ layered object, with refractive coefficients $n_0$, $n_1$, $n_2$, … $n_N = 1$ (which is empty space), using the refraction law

$$\sin\beta = \sin\alpha \cdot \frac{n_N}{n_{N-1}} \cdot \ldots \cdot \frac{n_2}{n_1} \cdot \frac{n_1}{n_0} = \sin\alpha \cdot \frac{1}{n_0}$$

$$\sin\alpha = n_0 \cdot \sin\beta = 1{,}000292 \cdot \sin\beta$$

$\alpha = 13°38'7''$.

Note:
The few angular seconds shift seems to be negligible, but for measurements in astronomy you need greater accuracy, therefore the refraction of the atmosphere must be considered.

(b) The initial refractive coefficient is 1.000292.
The changed refractive coefficient is:

$$n = 1 + 0.000292 \cdot \frac{1{,}010}{1{,}014} \cdot \frac{273}{273+15} = 1{,}000276 \cdot$$

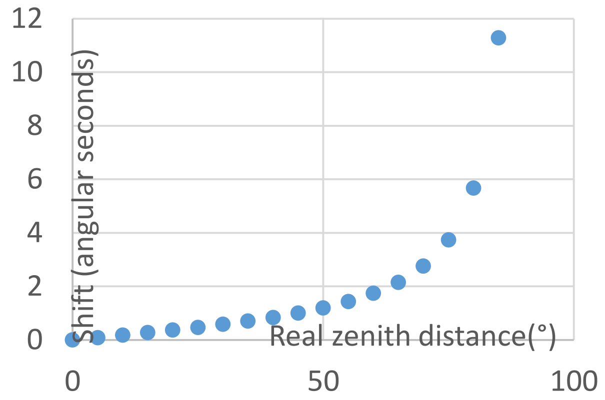
$$\sin\beta = \frac{\sin\alpha}{n} = \frac{\sin\alpha}{1{,}000276}$$

$\beta = 13°37'53''$.

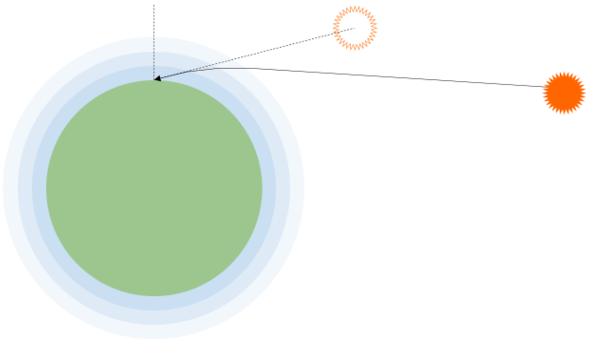
That is, the star seems to be farther from zenith by about 1 seconds, as before.

(c) $\delta = \alpha - \arcsin\left(\frac{\sin\alpha}{n}\right)$,

The following graph is obtained:




(d) The path of the light beam coming from near-horizon direction must not be seen as if passing parallel flat reflective surfaces:



# 4. Connecting the knowledge acquired in various chapters of physics teaching

## INTRODUCTORY EXERCISES

**4.1** In his novel entitled *Journey to the Moon* (1865) *Jules Verne* imagined a moon travel so that astronauts sit in the inside of a projectile shot from an immensely big canon.
(a) If you neglect aerodynamic resistance, to rotation of the Earth and the orbiting of the Moon, what would be the speed by which you ought to launch a cannonball to get it to the Moon?

(b) Let's assume that the mass of the projectile was 2000 kg. If the amount of energy released by the explosion of a ton of TNT is 4700 MJ, how many tons of TNT would be needed to fire that canon?

(c) If the gun barrel of the imagined canon is 500 metres long, what would be the acceleration the travellers had to endure?

**4.2** (a) How much the material thrown up on the surface of the Sun had to be accelerated in order to leave the Sun?

(b) The density of the white dwarf stars is in the magnitude of $10^9$ kg/m$^3$. What is the velocity of escape from the surface of a white dwarf with a mass equal to that of the Sun?

A meteoroid initially seen as quiescent arrives from the interplanetary space to the Solar system (a piece of rock).
(c) The meteoroid starts to fall towards the Sun as a result of the gravitational attraction of the Sun on a straight course. What will be the speed by which it hits the Sun?

(d) What will be the speed it accelerates to if it arrives on a parabolic course and the perihelion is found in a distance of the Earth? (Neglect the impact of the Earth and the other planets.)

(e) What is the speed it clashes into the Earth's atmosphere when hitting it "in front", or "from behind"?

**4.3** (a) The diameter of the Milky Way system is approximately 30 kpc (1 pc = 3.26 light year ). The angular diameter of the Abell 2218 galaxy cluster situated in a distance of 760 Mpc is 9.0 angular minutes. How many times is greater the diameter than that of the Milky Way system?

(b) Taking a photo of the galaxy cluster you can state that it consists of approximately 120 galaxies, like our Milky Way system. Estimate the mass of matter in the cluster.

(c) According to our measurements one of the galaxies, found 1000 kpc away from the middle of the cluster, moves with a speed of 950 km/s relative to this centre. Could the galaxy leave the cluster, if it had been only visible matter?

(d) Based on other measurements you arrive at the conclusion that the galaxy can not tear away from the gravitational field of the cluster, the 950 km/s velocity can be regarded as the rotational speed around the centre of the cluster. Based on this information, how much is the total mass of the cluster?

# 4. Connecting the knowledge acquired in various chapters of physics teaching

## GASES IN THE GRAVITATIONAL SPACE

**4.4** (a) Let's give an estimate to the average velocity of an oxygen molecule in the atmosphere, and compare is with the escape speed.

(b) What is the case with the hydrogen molecule?

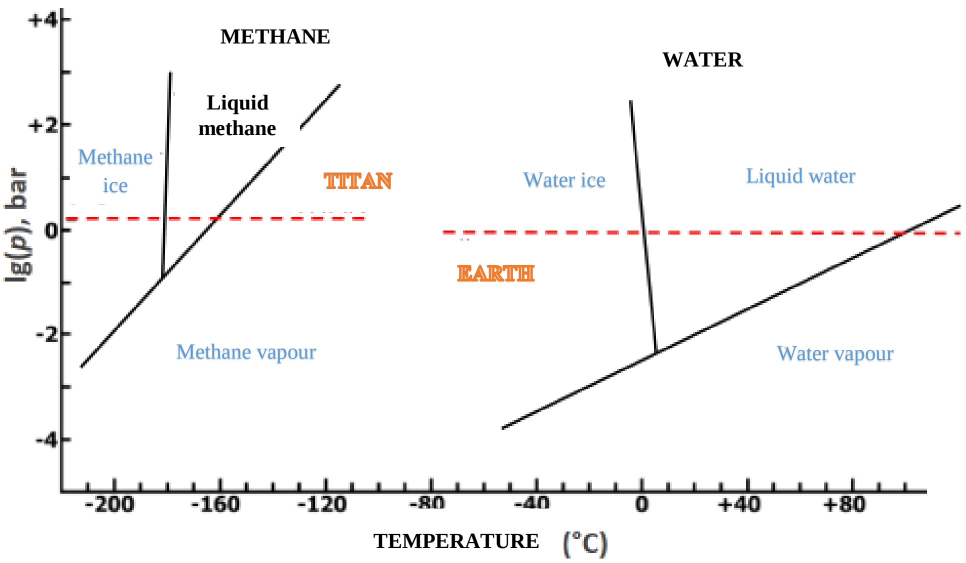
(c) Due to changes in the days and nights, temperature on Mars varies in the −83°C and −33°C range. Can carbon-dioxide be retained permanently in the Mars atmosphere?

(d) Mercury practically has no atmosphere. Let's assume that the average speed of a gas molecule can be maximum one tenth of the escape speed, otherwise it would leak out from the planet over millions of years. Assuming a mean temperature of 620 K on the sunny side of Mercury, what would be the molar mass of the gases which it could retain had it an atmosphere in the first place?

**4.5** (a) The moon called Titan of Saturn is the single moon in the Solar System which has a significant atmosphere. The mass of Titan is $1.34 \cdot 10^{23}$ kg. What is the molar mass of the gases which are able to stay in the Titan's atmosphere, provided we assume that the average speed of a gas molecule can be maximum one tenth of the escape speed, otherwise it would leak out from the planet over millions of years? Let's assume further that the temperature on Titan is roughly the same as on Saturn: about 95 K.

(b) The average 95 K temperature on Titan is obviously uninhabitable for our life forms. However, according to certain concepts it may provide some guidance to the scientists looking for the creation of life. In fact, Titan has an atmosphere consisting predominantly of nitrogen, but it also contains a substantial amount of methane. According to the measurements of space probes the Titan' atmosphere consists of a number of layers (including one absorbing UV-radiation), and methane clouds can be found in the lowermost stratum.

The figure below shows the so called phase diagram of methane and of water, indicating the air pressure prevailing on Titan and on Earth.
Why is it considered that methane may play a similar role on Titan as water does on Earth?




(c) The radius of Titan is 2600 km, and the thickness of the atmosphere is low in relation to the radius. Roughly consider the atmosphere as homogeneous in density. Provide estimate to the thickness of the atmosphere expressing surface air pressure in two ways.

**4.6** The thickness of the Earth's atmosphere is more than 100 km, while that of the neutron star named Cassiopeia-A discovered by the Chandra X ray telescope is merely a few centimetres. How can be that a small planet has a thicker atmosphere than a large object of this mass?

If temperature is seen as constant, the pressure and density of an atmosphere in the state of hydrostatic equilibrium would be reduced exponentially according to the so called barometric height formula:

$$p(h) = p_0 e^{-\frac{h}{H}},$$

$$\rho(h) = \rho_0 e^{-\frac{h}{H}}.$$

$H$ is the so called scale height: the level different at which density is dropped to its e-th part. The value of the scale height is

$$H = \frac{kT}{mg},$$

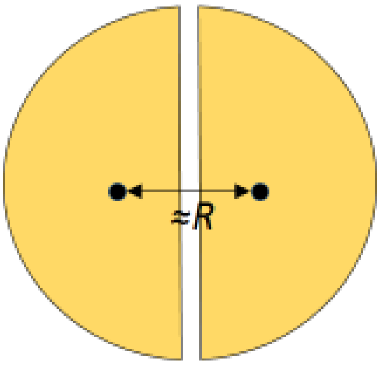
where $T$ stands for absolute temperature, $m$ being the average mass of the particles building up the atmosphere, and $k$ is the Boltzmann-constant.

(a) Gravitational acceleration on the surface of a neutron star is 100 billion times larger than it is on Earth. The temperature rises up to 3 million K, while on Earth it is only 240 K in average. Earthly atmosphere is a mixture of nitrogen and oxygen, 29 g/mol in average, while the atmosphere of a neutron star consists of carbon (12 g/mol).
What is the scale height on Earth and on a neutron star, respectively?

(b) In which distance from the surface will the density drop to one millionth of the surface value on Earth and on a neutron star, respectively?

**4.7** The Sun is a stable star: its huge mass does not fall into the centre of gravity as a result of the gravitational attraction, and its inner pressure does not blow it up, either. The gravitational and pressure related forces are in balance. Provide a rough estimate (in order of magnitude) to the temperature prevailing inside the Sun.

In order to do this, cut the Sun in half mentally. The distance between the centre of mass of the two hall Suns can be approached by the radius of the Sun:




(a) Imagine the mass of the parts into the centres of masses, how bit gravitational attraction is exerted on each other by the two half Suns?

(b) What is the average gravitational pressure on the surface separating the halves?

(c) The gravitational pressure is balanced by the pressure compiled from the gas pressure and the radiation pressure of the photons flowing outwards. In the case of stars with masses like that of the Sun gas pressure is dominating, you only need to deal with this. Seeing the Sun as a homogeneous density ideal gas ball, express ideal gas pressure as a function of density, molar mass and temperature.

(d) The gas consists of hydrogen nuclei (protons), electrons and helium nuclei. Take 0.75 grams as the average molar mass. What should be the temperature inside the Sun in order of magnitude?

# 4. Connecting the knowledge acquired in various chapters of physics teaching

## RADIATION PRESSURE

**4.8** The radiation intensity of the Sun in Earth distance is $S$ = 1370 W/m$^2$ (solar constant).
(a) How much is the total momentum of the incident photons on a 1 m$^2$ surface perpendicular to the direction of sunshine in one second?

(b) How much is the pressure of the Sun's radiation to a perpendicular surface in the distance of the Earth if it is assumed that all photons are absorbed?

(c) How much is the pressure of the Sun's radiation to a perpendicular surface in the distance of the Earth if it is assumed that all photons are reflected?

**4.9** (a) How much is the force exerted by the Sun 's radiation pressure on the Earth?

(b) Compare with the gravitational force exerted by the Sun on the Earth.

(c) What is the size of the body with the density of the Earth in the distance of the Earth, which is exposed to a radiation pressure from the Sun in the same order of magnitude as the gravitational attraction?

(d) Demonstrate that the ratio of the two forces was independent from the Sun distance in case of a given size.

**4.10** The dust tail of the comets is originated from the microscopic dust particles constituting the coma of the comet as a result of the radiation pressure of sunlight. (The wider, whitish light tail on the picture.)

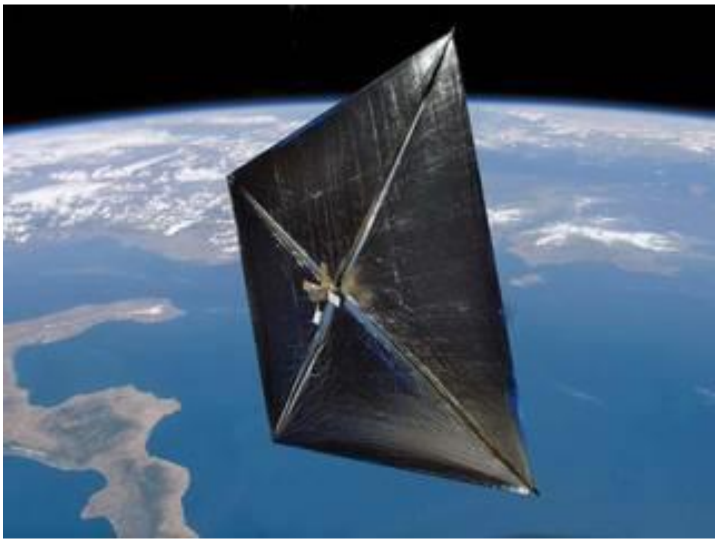
https://courses.lumenlearning.com/astronomy/ The Hale-Bopp comet

The Halley-comet had a perihelion on 9 February 1986 last time, orbiting in a distance of $9.0\cdot10^{10}$ m from the Sun.
(a) What was the radiation pressure the Sun exerted on it, if you neglect the light reflected by the comet as a rough approximation?

(b) Compare the Sun's radiation pressure and the forces exerted on the dust particle found in the dust tail of the comet, if the density of the particle is 2000 kg/m$^3$, and its shape is like a sphere with a diameter of 0.5 μm.

**4.11** The radiation pressure can be used to accelerate spacecrafts without the use if fuel. One way of doing so can be the so called solar sail the functional feasibility of which has been demonstrated by successful experiments. (In 2010 for the first time, by the Japanese satellite Icaros travelling to Venus.)

In the LightSail programme of NASA supported by civil funding solar sails are used to put satellites weighing merely 5 kg and serving various research goals to Earth orbit (CubeSat). The sail consists of thin light reflecting Mylar film with a useful surface of 32 $m^2$.



https://www.nasa.gov/mission_pages/tdm/solarsail/index.html

The image below shows the two CubeSat satellites released from the International Space Station. The sails are spread only when the satellite got to an appropriate distance from the space station.

https://phys.libretexts.org/TextBooks_and_TextMaps/University_Physics/

(a) Calculate the acceleration of the CubeSat satellite caused by the radiation pressure of sunshine.

(b) What would be the final speed of the spacecrafts if it had been accelerated this way for a year?

# Solution 4.

**4.1** (a) Seeing the Earth and the Moon as point masses in steady state, the projectile must pass at the point where the resulting gravitational space of the Earth and the Moon is zero, from there it will be accelerated towards the Moon. The mass of the Moon is approximately 1/81 times less than that of Earth, therefore this point is in the decimation point of the distance between the centres (towards the Moon). If the distance of the Moon is $r$ = 380 000 km, then (neglecting the attraction of the Moon at launch)

$$\tfrac{1}{2}mv^2 - \frac{\gamma mM}{R} = 0 - \frac{\gamma mM}{0{,}9r} - \frac{\gamma mM/81}{0{,}1r}$$

$$\tfrac{1}{2}v^2 = \frac{\gamma M}{R} - \frac{\gamma M}{r}\left(\frac{1}{0{,}9} + \frac{1}{8{,}1}\right)$$

$$v^2 = 2\gamma M\left(\frac{1}{R} - \frac{100}{81r}\right) =$$

$$= 2\cdot 6{,}67\cdot 10^{-11}\cdot 6{,}0\cdot 10^{24}\cdot$$

$$\cdot\left(\frac{1}{6{,}4\cdot 10^6} - \frac{100}{81\cdot 3{,}8\cdot 10^8}\right)$$

$v = 1.1\cdot 10^4$ m/s.

(At this level of accuracy the same figure is obtained when the Moon's gravitation is omitted entirely, since this is the known value of the escape speed.)

(b) $\tfrac{1}{2}mv^2 = 1{,}2\cdot 10^{11}\,\mathrm{J}$, corresponding to 26 tons of TNT.

(c) $v^2 = 2as$

$$a = \frac{v^2}{2s} = \frac{(1{,}1\cdot 10^4)^2}{2\cdot 500} = 1{,}2\cdot 10^5\,\mathrm{m/s^2} = 12000g$$

**4.2** (a) $\frac{1}{2}mv^2 - \frac{\gamma mM}{R} = 0 + 0$

The escape speed is $v = \sqrt{\frac{2\gamma M}{R}} =$

$$= \sqrt{\frac{2\cdot 6{,}67\cdot 10^{-11}\cdot 1{,}99\cdot 10^{30}}{6{,}96\cdot 10^8}} = 618\ \mathrm{km/s}.$$

(b) $\rho = \frac{3M}{4\pi R^3}$

$$\frac{R}{M} = \left(\frac{3}{4\pi\rho M^2}\right)^{1/3}$$

$$v = \sqrt{\frac{2\gamma M}{R}} = \sqrt{2\gamma\cdot\left(\frac{4\pi\rho M^2}{3}\right)^{1/3}}$$

$$v = \sqrt{2\cdot 6{,}7\cdot 10^{-11}\cdot\left(\frac{4\pi\cdot 10^9\cdot(2\cdot 10^{30})^2}{3}\right)^{1/3}}$$

$v = 6000\,\mathrm{km/s}$

(c) The same, as the escape speed on the surface of the Sun:

$$v = \sqrt{\frac{2\gamma M}{R}} = \sqrt{\frac{2\cdot 6{,}67\cdot 10^{-11}\cdot 2{,}0\cdot 10^{30}}{7{,}0\cdot 10^8}} =$$

$$= 6{,}2\cdot 10^5\,\mathrm{m/s} = 620\,\mathrm{km/s}$$

(d) $\frac{1}{2}mv^2 - \frac{\gamma mM}{r} = 0 + 0$

$$v = \sqrt{\frac{2\gamma M}{r}} == \sqrt{\frac{2\cdot 6{,}67\cdot 10^{-11}\cdot 1{,}99\cdot 10^{30}}{1{,}5\cdot 10^{11}}} =$$

= 42 km/s.

(e) The escape speed of the Earth on its course is approximately 30 km/s. That is, the impact speed is 72 km/s and 12 km/s coming from front or from the back, respectively.

**4.3** (a) 9.0' = $2.6\cdot 10^{-3}$ rad,

760Mpc = $7.6\cdot 10^5$ kpc.

The size of the cluster is

$$D = d\cdot\alpha = 7{,}6\cdot 10^5\cdot 2{,}6\cdot 10^{-3} = 2000\,\mathrm{kpc}$$

This is about 70 times the diameter of the Milky Way system.

(b) The Milky Way system consists of approximately 100 billion stars. The mass of the Sun (an average star) is $2\cdot 10^{30}$ kg,
thus, the total mass of 120 such galaxies will be
$120\cdot 10^{11}\cdot 2\cdot 10^{30} = 2.4\cdot 10^{43}$ kg

(c) The galaxy is found at the edge of the cluster. Assuming, that the galaxy had a spherical symmetry, the escape speed will be

$$v = \sqrt{\frac{2\gamma M}{R}} = \sqrt{\frac{2\cdot 6{,}7\cdot 10^{-11}\cdot 2{,}4\cdot 10^{43}}{1000\cdot 3{,}1\cdot 10^{19}}} = 320\,\mathrm{km/s}$$

The velocity of the galaxy is a lot more, it could get loose.

(d) The galaxy is found at the edge of the cluster, therefore the mass found inside of its orbit is practically the total mass of the galaxy.

$$\frac{v^2}{R} = \frac{\gamma M}{R^2}$$

$$M = \frac{v^2 R}{\gamma} = \frac{(9{,}5\cdot 10^5)^2 \cdot 1000 \cdot 3{,}1\cdot 10^{19}}{6{,}7\cdot 10^{-11}}$$

$$M = 4{,}2\cdot 10^{44}\,\text{kg}$$

(about 20 times of the visible mass).

**4.4** (a) According to the kinetic gas theory the average kinetic energy of the molecules at $T$ temperature is:

$$\frac{1}{2}mv^2 = \frac{3}{2}kT\,.$$

Taking the average temperature as 10°C:

$$v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\cdot 1{,}38\cdot 10^{-23}\cdot 283}{0{,}032/6\cdot 10^{23}}} =$$

$$= \sqrt{\frac{3\cdot 1{,}38\cdot 6\cdot 283}{0{,}032}} = 470\text{m/s}$$

The escape speed on the surface of the Earth:

$$v = \sqrt{\frac{2GM}{R}} =$$

$$= \sqrt{\frac{2\cdot 6{,}67\cdot 10^{-11}\cdot 6{,}0\cdot 10^{24}}{6{,}4\cdot 10^6}} = 11\text{km/s}$$

The velocity of an oxygen molecule is approximately 4%, very few molecules move fast enough to escape, the Earth is able to retain oxygen in its atmosphere.

(b) The mass of the hydrogen molecule is one 16th of that of the oxygen molecule, that is its velocity is four times greater, about a sixth of the escape speed: hydrogen from the atmosphere slowly escapes.

(c) The escape speed on Mars: $v = \sqrt{\frac{2GM}{R}} =$

$$= \sqrt{\frac{2\cdot 6{,}67\cdot 10^{-11}\cdot 6{,}42\cdot 10^{23}}{3{,}40\cdot 10^6}} = 5{,}02\text{km/s}$$

The molar mass of carbon-dioxide is 44.0 grams. The upper temperature is $T$ = 240 K. The quadratic mean velocity of the carbon-dioxide molecule is

$$v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\cdot 1{,}38\cdot 10^{-23}\cdot 240}{0{,}0440/6{,}02\cdot 10^{23}}} =$$

$$= \sqrt{\frac{3\cdot 1{,}38\cdot 6{,}02\cdot 240}{0{,}0440}} = 369\text{m/s}$$

Less than one tenth of the escape speed, the carbon-dioxide can stay permanently. (The atmosphere of the Mars consists mainly of carbon-dioxide.)

(d) $\sqrt{\frac{3kT}{m}} < 0{,}1\sqrt{\frac{2GM}{R}}$

$$100\cdot\frac{3kT}{m} < \frac{2GM}{R}$$

$$m > \frac{150kTR}{GM} =$$

$$= \frac{150\cdot 1{,}38\cdot 10^{-23}\cdot 620\cdot 2{,}4\cdot 10^6}{6{,}67\cdot 10^{-11}\cdot 3{,}3\cdot 10^{23}} = 1{,}4\cdot 10^{-25}\,\text{kg}$$

This corresponds to 84 g molar mass. Mercury could retain atmospheric gases with at least such a molar mass.

Note:

Due to its small mass and high surface temperature Mercury is not able to retain any significant atmosphere. Yet, recent space probe observations suggest a very rare atmosphere, thin on the sunny side and thicker, tail-like on the night side. It is originated mainly from the permanent (micro)meteorite impacts, surface materials on the impact site and the sun wind. As it is generated, this atmospheric matter escapes into the space, yet the aforementioned processes re-produce it all the time. Its composition is about 40% oxygen, 30% sodium, 20% hydrogen, 5% helium, but potassium, xenon, krypton, argon, neon, nitrogen, water and carbon-dioxide, calcium and magnesium were also detected in it. According to the exercise, krypton and xenon could stay permanently, but this rare atmosphere is not like the one on Earth, where the atmospheric particle collide to each other continuously. Particles in the Mercury atmosphere do not meet at all before escaping in space.

**4.5** (a) $\sqrt{\frac{3kT}{m}} < 0{,}1\sqrt{\frac{2GM}{R}}$

$$100\cdot\frac{3kT}{m} < \frac{2GM}{R}$$

$$m > \frac{150kTR}{GM}$$

$$m > \frac{150\cdot 1{,}38\cdot 10^{-23}\cdot 95\cdot 2{,}575\cdot 10^6}{6{,}67\cdot 10^{-11}\cdot 1{,}34\cdot 10^{23}}$$

$$m > 5{,}65\cdot 10^{-26}\,\text{kg}$$

This equals 34 g molar mass.

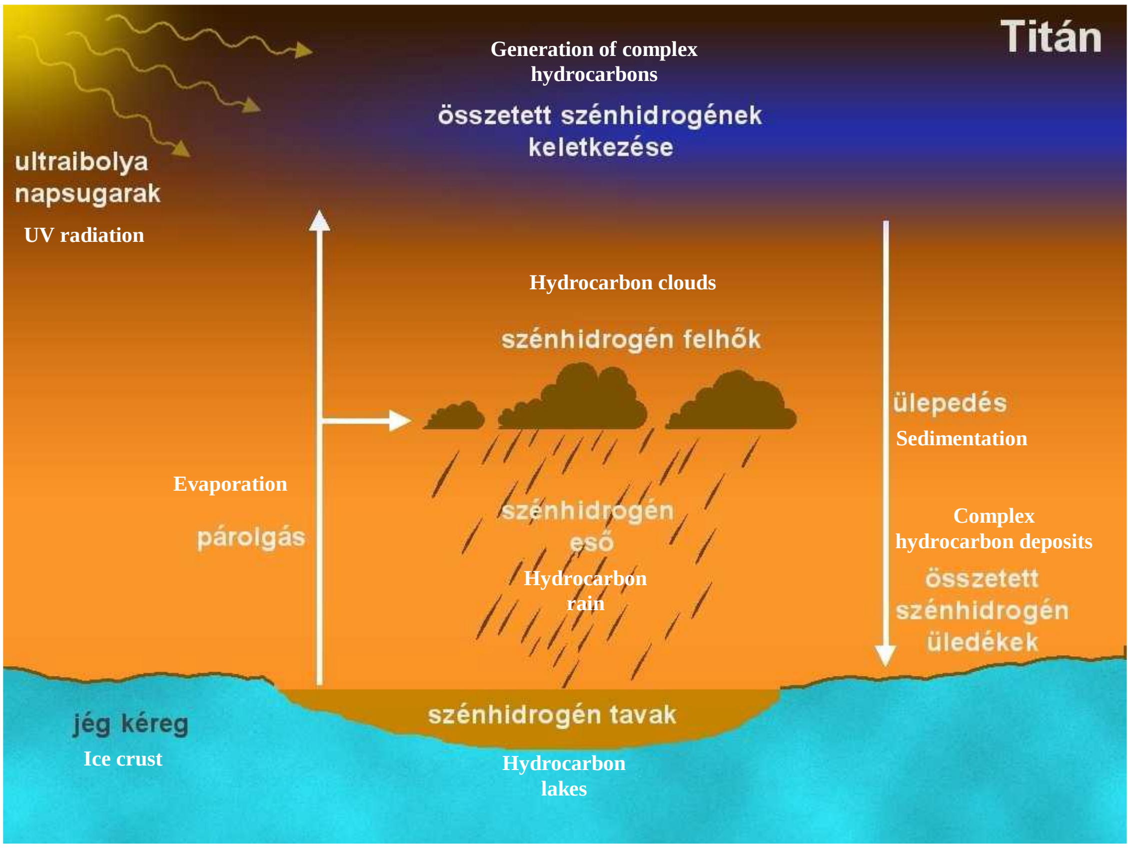
Note:
The Titan’s atmosphere consists of 90% nitrogen. Ammonium ice evaporated by the heat originating from radioactive decay is broken up to nitrogen and hydrogen by the Sun’s UV radiation. The nitrogen thus developed feeds the atmosphere.

(b) The pressure and temperature on Titan is close to the triple point of methane, just as the pressure and temperature on Earth is close to the triple point of water. Thus methane can be found in all three states on it, as water on Earth. (Methane snow may fall on the polar caps, methane rain may feed methane rivers, lakes, in warmer regions. See the figure on the next page.)

(c) Based on the hydrostatic correlation, the pressure on the surface is

$$p = \rho g h \text{, where}$$

$$g = \frac{\gamma M}{r^2} = \frac{6{,}7 \cdot 10^{-11} \cdot 1{,}3 \cdot 10^{23}}{(2{,}6 \cdot 10^6)^2} = 1{,}3 \frac{\text{m}}{\text{s}^2}$$




http://www.csillagasz.at/

On the other hand, according to the state formula of the ideal gas

$$p = \frac{nRT}{V} = \frac{\rho RT}{\mu},$$

where $\mu$ is the molar mass.

$$\rho g h = \frac{\rho RT}{\mu}$$

The temperature is 95 K, and the atmosphere consists mainly of nitrogen.

$$h = \frac{RT}{\mu g} = \frac{8{,}3 \cdot 95}{0{,}028 \cdot 1{,}3} \approx 20\text{km}$$

Note:
Since the density and pressure of the atmosphere declines with altitude, the height thus calculated is in fact the altitude where decline is $1/e$ times less, see the next exercise.

**4.6** (a) On the Earth

$$H = \frac{kT}{mg} = \frac{1{,}38 \cdot 10^{-23} \cdot 240}{0{,}029/(6 \cdot 10^{23}) \cdot 9{,}8} =$$

$$= \frac{1{,}38 \cdot 240 \cdot 6}{0{,}029 \cdot 9{,}8} = 7\text{km}$$

On a neutron star:

$$H = \frac{kT}{mg} == \frac{1{,}38 \cdot 3 \cdot 10^6 \cdot 6}{0{,}012 \cdot 10 \cdot 10^{11}} = 2\text{mm}.$$

(b) ln 1 000 000 ≈ 14.
Fourteen times the scale height on the Earth will be 97 km, on a neutron star approximately 2.8 cm.

**4.7** (a) $F_{\text{grav}} = \frac{\gamma \cdot (M/2)^2}{r^2} =$

$$= \frac{6{,}7 \cdot 10^{-11} \cdot (1 \cdot 10^{30})^2}{(7{,}0 \cdot 10^8)^2} = 1{,}4 \cdot 10^{32}\,\text{N}$$

(b) $p_{grav} = \frac{F_{grav}}{r^2\pi} = \left( \frac{\gamma \cdot (M/2)^2}{r^4 \cdot \pi} = \right)$

$$= \frac{1{,}4 \cdot 10^{32}}{(7{,}0 \cdot 10^8)^2 \cdot \pi} = 8{,}9 \cdot 10^{13}\,\text{Pa}\,.$$

(c) $p = n \cdot \frac{RT}{V} = \frac{M}{m} \cdot \frac{RT}{V} = \frac{\rho RT}{m}$,

where $R$ is the universal gas constant.

(d) $T = \frac{m \cdot p}{\rho \cdot R} = \frac{4\pi \cdot r^3 \cdot m \cdot p}{3M \cdot R} =$

$$= \frac{4\pi \cdot r^3 \cdot m}{3M \cdot R} \cdot \frac{\gamma \cdot (M/2)^2}{r^4 \cdot \pi} = \frac{\gamma mM}{3R \cdot r} =$$

$$= \frac{6{,}7 \cdot 10^{-11} \cdot 0{,}00075 \cdot 2 \cdot 10^{30}}{3 \cdot 8{,}3 \cdot 7{,}0 \cdot 10^8} = 6\text{millió}\,\text{K}$$

The temperature is in the 10 million K range.

**4.8** (a) The total energy of the photons is 1370 J, and the momentum of a photon with $E$ energy level is $E/c$, therefore the total momentum transmitted to a unit surface during a unit time is

$$\frac{S}{c} = \frac{1370 \frac{\text{J}}{\text{s} \cdot \text{m}^2}}{3{,}00 \cdot 10^8 \frac{\text{m}}{\text{s}}} = 4{,}57 \cdot 10^{-6}\,\frac{\text{N}}{\text{m}^2}$$

(b) The momentum transmitted to a unit surface during a unit time is the force acting on the unit surface, in other words the pressure. If a photon is absorbed, the transferred momentum equals with that of the photons, hence

$$p = 4{,}57 \cdot 10^{-6}\,\frac{\text{N}}{\text{m}^2}\,.$$

(c) If photons are reflected, the of their momentums is twice the original momentum, therefore

$$p = 2 \cdot 4{,}57 \cdot 10^{-6} = 9{,}13 \cdot 10^{-6}\,\frac{\text{N}}{\text{m}^2}\,.$$

**4.9** (a) The solar constant $S$ = 1400 W/m$^2$. The momentum of a photon with $E$ energy is $E/c$. The force on surface $A$ if the total momentum transmitted within a unit of time. If reflection is disregarded, this force is the total momentum of the photons incident on the cross section of the Earth in 1 second:

$$F = \frac{A \cdot S}{c} = \frac{R^2\pi \cdot S}{c} =$$

$$= \frac{(6{,}4 \cdot 10^6)^2 \cdot \pi \cdot 1400}{3{,}0 \cdot 10^8} \frac{\text{Ws}}{\text{m}} = 6 \cdot 10^8\,\text{N}\,.$$

(b) $F = \frac{\gamma mM}{r^2} =$

$$= \frac{6{,}7 \cdot 10^{-11} \cdot 6{,}0 \cdot 10^{24} \cdot 2{,}0 \cdot 10^{30}}{(1{,}5 \cdot 10^{11})^2} = 3{,}6 \cdot 10^{22}\,\text{N}$$

14 orders of magnitude larger.

(c)

$$\frac{R^2\pi \cdot S}{c} = \frac{\gamma mM}{r^2}$$

$$\frac{R^2\pi \cdot S}{c} = \frac{\gamma \cdot 4R^3\pi \cdot \rho \cdot M}{3r^2}$$

$$\frac{S}{c} = \frac{\gamma \cdot 4R \cdot \rho \cdot M}{3r^2}$$

$$R = \frac{3r^2 S}{4\gamma\rho Mc}$$

$$R = \frac{3 \cdot (1{,}5 \cdot 10^{11})^2 \cdot 1400}{4 \cdot 6{,}7 \cdot 10^{-11} \cdot 5500 \cdot 2 \cdot 10^{30} \cdot 3 \cdot 10^8}$$

$$R = 1 \cdot 10^{-7}\,\text{m} = 0{,}1\mu\text{m}$$

(d) The solar constant, and proportionally the radiation pressure, decline in an inverse ratio with the square of the sun distance, just like gravitational attraction does. The ratio of the two forces is thus constant.

**4.10** (a) The radiation pressure is larger in inverse proportion with the square of the distance, than it is in Earth distance:

$$p = \frac{S}{c} \cdot \left(\frac{15}{9}\right)^2 = 4{,}6 \cdot 10^{-6} \cdot \left(\frac{15}{9}\right)^2 = 1{,}3 \cdot 10^{-5}\,\frac{\text{N}}{\text{m}^2}$$

(b) The force exerted by the radiation pressure is:

$$F = R^2\pi \cdot p =$$

$$= (2{,}5 \cdot 10^{-7})^2 \cdot \pi \cdot 1{,}3 \cdot 10^{-5} = 3 \cdot 10^{-18}\,\text{N}\,.$$

(Due to reflection, it is somewhat larger in reality.)

The Sun’s gravitational force:

$$F = \frac{\gamma mM}{r^2} = \frac{\gamma \cdot 4R^3\pi \cdot \rho \cdot M}{3r^2} =$$

$$= \frac{6{,}7 \cdot 10^{-11} \cdot 4(2{,}5 \cdot 10^{-7})^3 \pi \cdot 2000 \cdot 2{,}0 \cdot 10^{30}}{3 \cdot (9{,}0 \cdot 10^{10})^2} =$$

$$= 2 \cdot 10^{-18}\,\text{N}$$

Note:
If you investigate the development of the tail, you have to taken into account the gravitation of the comet nucleus to determine acceleration, and the impact of the dust and gas cloud surrounding the nucleus.

**4.11** (a) The radiation pressure in Earth distance:

$$\frac{2S}{c}=\frac{2\cdot 1370\,\frac{\mathrm{J}}{\mathrm{s}\cdot\mathrm{m}^2}}{3{,}00\cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}}=9{,}13\cdot 10^{-6}\,\frac{\mathrm{N}}{\mathrm{m}^2}.$$

The force exerted on the sail

$$F=pA=9{,}13\cdot 10^{-6}\cdot 32=2{,}9\cdot 10^{-4}\,\mathrm{N}.$$

The acceleration caused:

$$a=\frac{F}{m}=\frac{2{,}9\cdot 10^{-4}}{5}=5{,}8\cdot 10^{-5}\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

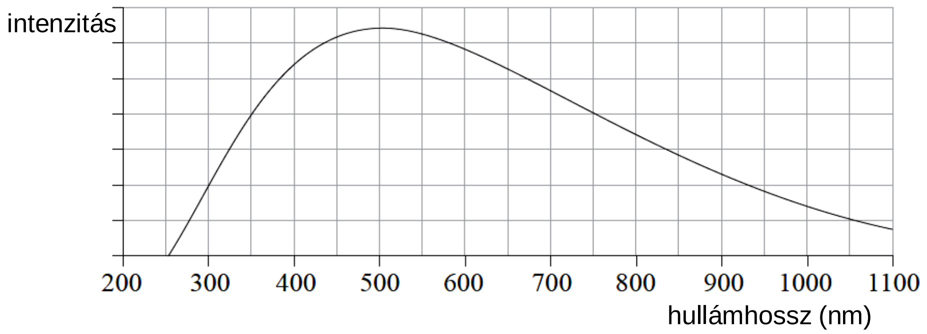
(b) $v=at=5{,}8\cdot 10^{-5}\cdot 365\cdot 24\cdot 3600=$

$$=1800\,\frac{\mathrm{m}}{\mathrm{s}}=6600\,\frac{\mathrm{km}}{\mathrm{h}}$$

# 5. The application of the laws of thermal radiation
## DISPLACEMENT LAW

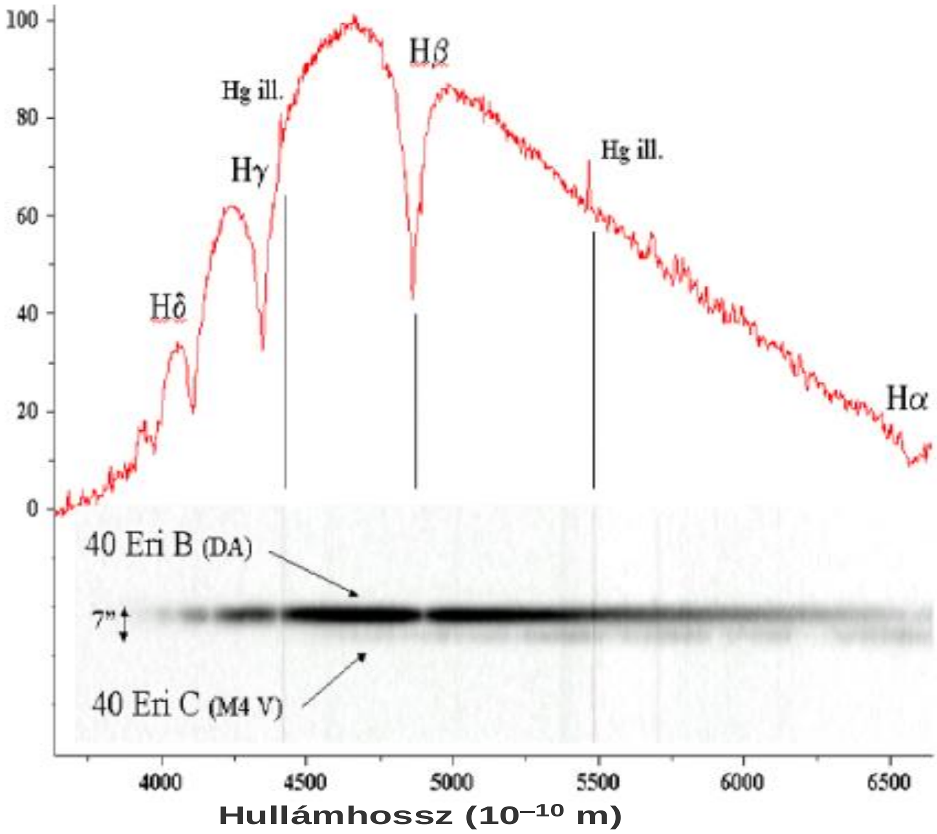
**5.1** The figure shows the Sun's black body spectrum. Determine the effective temperature of the Sun on the basis of the figure (in other words, the corresponding black body temperature).

intensity




wave length

**5.2** The figure shows the spectrum of the white dwarf 40 Eridani B. How much is the 40 Eridani B's temperature?




http://spacemath.gsfc.nasa.gov

**5.3** Deneb is the brightest star of the Cygnus constellation (α Cygni). Effective temperature is 10 500 K.

Antares is a super giant star in the Scorpion constellation (α Scorpi). Effective temperature is 3000 K. In which colour we see them?

# 5. The application of the laws of thermal radiation

## LUMINOSITY AND INTENSITY

**5.4** The intensity of the radiation of a body radiating with *P* power at a place in a distance *d* from the body will be $I = \frac{P}{4\pi d^2}$. Which assumption(s) is this correlation built?

**5.5** The intensity of the Sun's radiation in the Earth distance is approximately 1400 W/m$^2$ (solar constant).
Why is it that the incident average intensity on the Earth surface (i.e. the top of the atmosphere) is merely 350 W/m$^2$.

**5.6** The Sun's radiation power (luminosity) is $3.9 \cdot 10^{26}$ W, its distance is $1.5 \cdot 10^{11}$ m. Demonstrate that the value of the solar constant is about 1400 W/m$^2$.

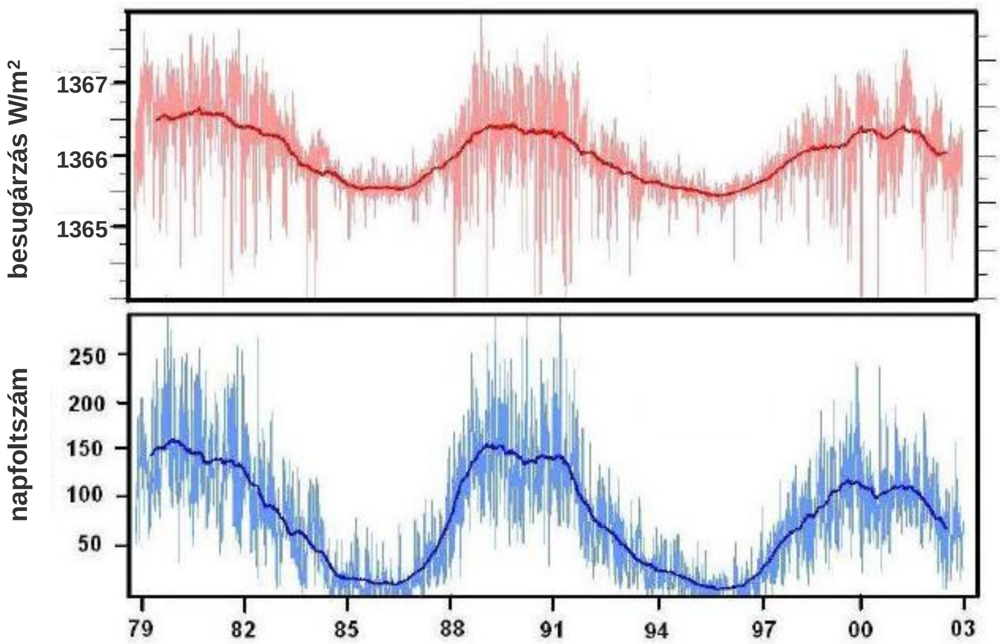
**5.7** The luminosity of the star called Ross 128 is $1.1 \cdot 10^{21}$ W, intensity of its light is $7.9 \cdot 10^{-15}$ W/m$^2$ in Earth distance. How many light years is its distance from Earth?

**5.8** The two graphs below show the changes in the incident solar radiation on Earth ( high atmosphere), and the number of sunspots throughout a 25 year period.
(The thin lines are daily fluctuations, thick lines show annual running averages.)
(a) What is the correlation between irradiation intensity and the number of sunspots according to the graphs?

(b) How much as the average intensity (solar constant)? What is the extent of intensity oscillation within a sunspot cycle?

(c) You mounted solar cells on your rooftop in 1985 and produced a total of 3000 kWh electric power across the years. Assuming the ground level intensity follows the changes in the high atmosphere, how much more energy can be produced by the same system in 1989?

number of sunspots, irradiation in W/m$^2$

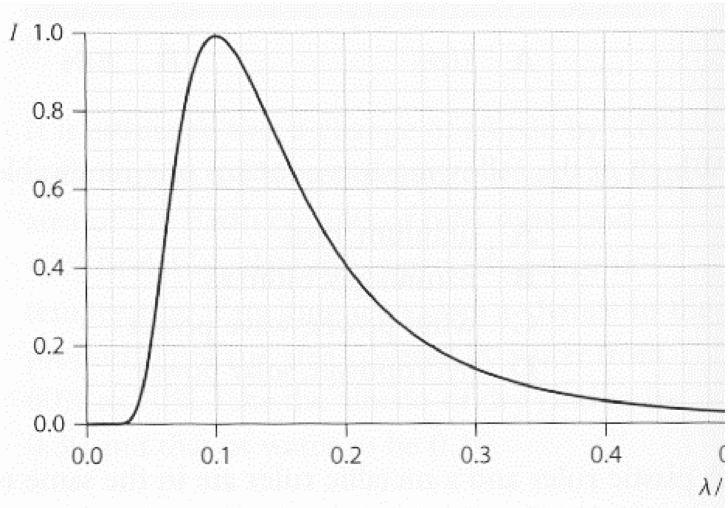



http://spacemath.gsfc.nasa.gov

# 5. Application of the regularities applicable to thermal radiation

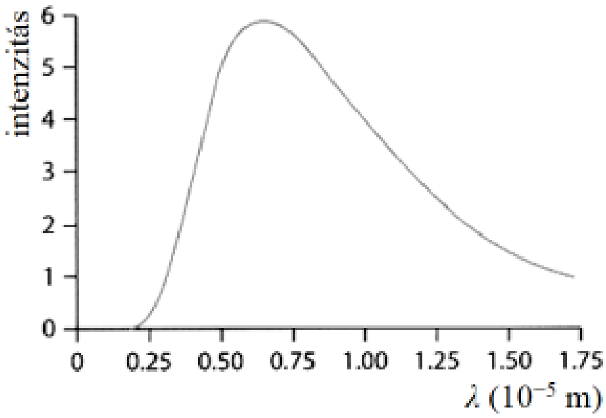
## STEFAN−BOLTZMANN-LAW

**5.9** (a) The graph shows the distribution of the radiation intensity of a black body according to its wavelength (arbitrary units). Draw the radiation instensity distribution of a body with half the temperature.




The graph below shows the distribution of the radiation intensity of a black body according to its wavelength.
(b) What is the temperature of the body?

(c) Draw the radiation distribution of a black body with a temperature of 300 K.




**5.10** (a) How many times greater emanation power is derived from a black body per unit area if its temperature is 900 K, as opposed to a temperature of 300 K ?

(b) How many times the effective radiated power grows to, if the temperature of a body is increased from 100°C to 150°C?

**5.11** Betelgeuse is the brightest star of the Orion constellation seen overleaf. Its luminosity is $3.9 \cdot 10^{30}$ W, the effective temperature is 3000 K.
(a) What is the radius of Betelgeuse?

(b) What is the wavelength on which the Betelgeuse has its maximum radiation intensity?

(c) What is the colour of Betelgeuse? Which could be it on the picture?

**5.12** Antares is the brightest star in the Scorpion constellation. It's a double system, the brighter star being Antares A, with an effective temperature of 3100 K. Its satellite, Antares B has an effective temperature of 15 000 K, and a luminosity (effective radiated power) of 1/40 of that of Antares A.
(a) How many times greater is the radius of Antares A, than that of Antares B?

(b) What is the colour of Antares?

**5.13** The effective radiated power of the Sun (Sun luminosity) is $3.90\cdot10^{26}$ W.
The Sun's radius is $6.96\cdot10^{8}$ m.
(a) How many metres square of the Sun's surface emanates an output of 1000 MW?

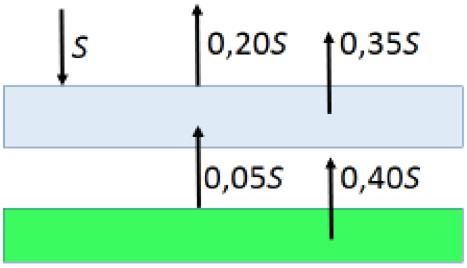
(b) How much is the temperature on the surface of the Sun based on the data provided?

(c) According to certain theoretical assumptions the Sun's temperature in an early stage of the formation of the Solar System was 5000 K, and the radius was 1.02 times that of today. How much must have been the solar constant? (The Earth orbital radius was the same.)

**5.14** Image the distance $d$ between Sun and Earth is diminishing. Earth $T$ temperature grows. The solar radiation ratio incident on Earth is proportional with $1/d^2$, while the effective radiated power of the Earth relates to $T^4$. The average temperature of the Earth is 288 K. How much this distance could rise if the distance had dropped 1%?

# 5. The application of the laws of thermal radiation
## ALBEDO, ESTIMATE OF TEMPERATURE ON PLANETS

**5.15** A certain ratio of incident radiation (planetary albedo) is reflected by the planets, the rest is absorbed. The intensity of the incident solar radiation on the surface of a planet is *S*. The figure shows the planet's energy balance: The intensity reflected by the atmosphere is 0.20*S*, intensity emanated by the atmosphere is 0.35*S*. The intensity reflected by the surface is 0.05*S*, intensity emanated by the surface is 0.40*S*. How much is the planet's albedo?




**5.16** Estimates suggest that a 0.01 drop in the albedo results in a 1°C temperature rise. The large surfaces on the Earth are made of water (60%) and firm land (40%). The albedo of the land is 0.3, that of the water is 0.1. Let's assume, the part covered by water grows from 60% to 70% as a result of melting ice. What will be the average temperature increase in the region?

**5.17** The average radiation intensity incident on the Earth surface is 340 W $m^{-2}$. The average planetary albedo is 0.30. Carbon-dioxide volumes grew substantially in the past decades in the Earth 's atmosphere. Estimates suggest doubling of the carbon-dioxide would reduce the Earth's albedo by 0.01. Based on this, how much will be the reduction in the intensity emanated into the space by Earth upon such doubling?

**5.18** (a) The solar radiation intensity in Earth distance is $S = 1380$ W/$m^2$.
The Earth's average albedo is $\alpha = 0.30$.
Demonstrate that the average radiation intensity reflected from the Earth surface is about 100 W/$m^2$.

(b) Explain why is the average intensity absorbed by the Earth $\frac{S(1-\alpha)}{4}$, and calculate its value.

(c) How much is the average radiation intensity emitted by the Earth surface?

**5.19** (a) The Sun's radiation power (luminosity) is $L = 3.90 \cdot 10^{26}$ W. In time average, how much is the incident solar energy within a unit time on a unit surface of a planet rotating around its own axis and orbiting around the Sun in a distance of *r* (in other words the incident intensity)?

(b) How much is the average incident intensity on the Earth, on Mercury, on Venus and on Mars?

(c) In case of a thermal equilibrium the planet would emanate energy to the space at the same power as the absorbed capacity. Let's assume that the planets behave like black bodies in terms of emanation. Calculate from the result in exercise (b) and the value of the planetary albedo taken from the table what is the temperature at which the four planets above eradiate into space as black bodies.

(d) Actually, the average surface temperatures are as follows:
Mercury : 170°C, Venus: 460°C, Earth : +15°C, Mars: −65°C. Which planets concur with the calculated values and which do not? What should be the reason?

# Solution 5.

**5.1** (a) The wave length associated with the maximum is 500 nm, thus $T = \frac{2{,}9 \cdot 10^{-3}}{5{,}0 \cdot 10^{7}} = 5800\text{K}$.

**5.2** The maximum of the curve is at 4650 angström, therefore $T = \frac{2{,}897 \cdot 10^{-3}}{4{,}65 \cdot 10^{-7}} = 6200\text{K}$

**5.3** Deneb: $\lambda_{\max} = \frac{2{,}9 \cdot 10^{-3}}{10500} = 280\,\text{nm}$ (ultraviolet), the visible range of the spectrum is perceived as blue.

Antares: $\lambda_{\max} = \frac{2{,}9 \cdot 10^{-3}}{3000} = 970\,\text{nm}$ (infrared), the visible range of the spectrum is perceived as red.

**5.4** Assume that radiation intensity is the same in all directions, and that no absorption occurs during the covering of the distance *d*.

**5.5** Earth with *R* radius is hit by a cylindrical $R^2\pi$ beam of radiation from the sun, which is distributed on the surface of the rotating Earth in a area $4R^2\pi$, in other words four times that size. The average intensity is thus 1400 W/m$^2$ /4 = 350 W/m$^2$.

**5.6** $S = \frac{L_{\text{Nap}}}{4\pi \cdot d^2} = \frac{3{,}9 \cdot 10^{26}}{4\pi \cdot (1{,}5 \cdot 10^{11})^2} = 1400 \frac{\text{W}}{\text{m}^2}$.

**5.7** From the $I = \frac{L}{4\pi d^2}$ interdependence of *L* luminosity, *d* distance and *I* intensity:

$$d^2 = \frac{1{,}1 \cdot 10^{21}}{4\pi \cdot 7{,}9 \cdot 10^{-15}},$$

where $d = 1.05 \cdot 10^{17}$ m = 11 light year .

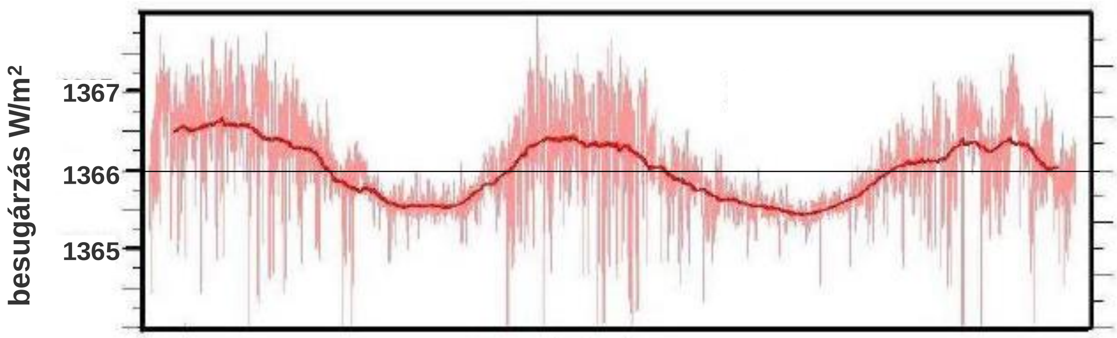
**5.8** (a) The intensity seems to follow the 11 years sunspot cycle. Intensity is higher when sunspots are more active.

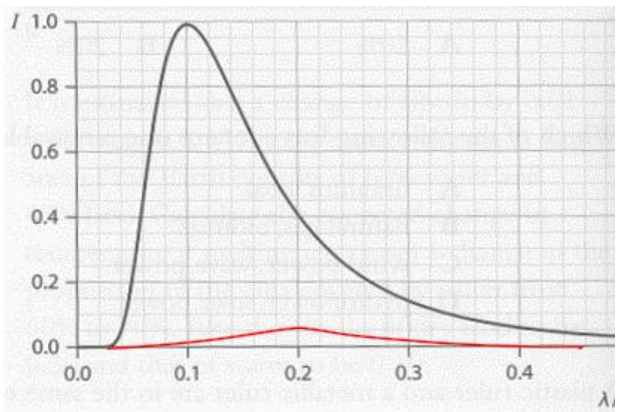
(b) The average is about 1366 W/m$^2$, the largest difference is approximately ±0.5 W/m$^2$.

(c) It was at minimum in 1985, and maximum in 1989, with a difference of approximately 1 W/m$^2$.

The additional production is $3000 \cdot \frac{1}{1366} \approx 2\text{kWh}$.

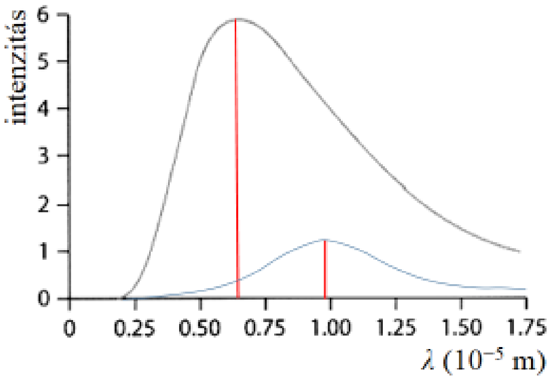
irradiation



**5.9** (a) Based on Wien's law of radiation the wavelength associated with maximum intensity is twice as large, while the area under the curve is $(1/2)^4$ = 1/16 times less according to the Stefan–Boltzmann-law.

(c) 300/450 = 0.67, the temperature is thus reduced to approximately 2/3 part, and the wavelength associated with the peak is one and a half times as much, approximately $0.97 \cdot 10^{-5}$ m. The area under the curve is reduced to approximately $(300/446)^4$ = 0.20 times, that is to one fifth.




**5.10** (a) $3^4$ = 81 times.

(b) $(423/373)^4$ = 1.65 times.

**5.11** (a) Using the Stefan–Boltzmann-law:

$$L = \sigma \cdot 4\pi R^2 \cdot T^4$$

$$R = \sqrt{\frac{L}{\sigma \cdot 4\pi \cdot T^4}} =$$

$$= \sqrt{\frac{3{,}9 \cdot 10^{30}}{5{,}67 \cdot 10^{-8} \cdot 4\pi \cdot 3000^4}} = 2{,}6 \cdot 10^{11}\,\text{m}$$

(b) $\lambda_{\max} = \dfrac{2{,}90 \cdot 10^{-3}}{3000} = 970\text{nm}$ .

(c) Red (giant). The upper left star on the image.

**5.12** (a) $\dfrac{L_A}{L_B} = \dfrac{\sigma \cdot 4\pi R_A^2 \cdot T_A^4}{\sigma \cdot 4\pi R_B^2 \cdot T_B^4} = 40$, of which

$$\frac{R_A}{R_B} = \sqrt{40}\left(\frac{15000}{3100}\right)^2 = 150 \text{ times as big.}$$

(b) The colour is determined by Antares A with 40 times as strong luminosity.

(b) The wavelength read from the graph associated with the maximum intensity is $0.65 \cdot 10^{-5}$ m, temperature from Wien's law:

$$T = \frac{2{,}90 \cdot 10^{-3}}{0{,}65 \cdot 10^{-5}} \approx 450\text{K} .$$

$$\lambda = \frac{2{,}9 \cdot 10^{-3}}{3100} = 9{,}35 \cdot 10^{-7}\,\text{m} = 935\text{nm}$$

The 935 nm is infra red, the colour of Antares is thus red (i.e. a red super giant).

**5.13** (a) On the surface of the Sun

$$I = \frac{3{,}90 \cdot 10^{26}}{4\pi \cdot (6{,}96 \cdot 10^8)^2} = 6{,}41 \cdot 10^7 \frac{\text{W}}{\text{m}^2} =$$

= 64.1 MW/m$^2$.

You need 15.6 m$^2$ for 1000 MW.

(b) $L = \sigma \cdot 4\pi R^2 \cdot T^4$, of which

$$T = \sqrt[4]{\frac{L}{\sigma \cdot 4\pi R^2}} =$$

$$\sqrt[4]{\frac{3{,}90 \cdot 10^{26}}{5{,}67 \cdot 10^{-8} \cdot 4\pi \cdot (6{,}96 \cdot 10^8)^2}} = 5800\text{K} ,$$

or directly derived from (a):

$$T = \sqrt[4]{\frac{6{,}41 \cdot 10^7}{5{,}67 \cdot 10^{-8}}} = 5800\text{K} .$$

(c) $1370 \cdot \left(\dfrac{5000}{5800}\right)^4 \cdot \left(\dfrac{1{,}02}{1}\right)^2 = 788 \dfrac{\text{W}}{\text{m}^2}$ .

**5.14** Upon a 1% reduction of the distance $d$ $1/d^2$ is increased by 2%, in other words the solar constant is 2% higher. When in balance, a 2% increase in incident radiation means the same increase in emanation. If $T^4$ is 2% higher, $T$ absolute temperature grows by 0.5%: 288·0.05 = 1.4 K.

Or:

Assuming an equilibrium, $T$ is proportional to $1/\sqrt{d}$, therefore $T' = \dfrac{T}{\sqrt{0.99}} = 289{,}5\text{K}$ , the temperature is therefore 1.5 K.

**5.15** The total reflected ratio:

0.20 + 0.05 = 0.25.

**5.16** The reflected ratio is originally

$$0{,}3\cdot 0{,}4+0{,}1\cdot 0{,}6=0{,}18.$$

The changed value:

$$0{,}3\cdot 0{,}3+0{,}1\cdot 0{,}7=0{,}16.$$

is 0.02 less meaning a 2°C rise in temperature.

**5.17** The reflected intensity is reduced:

$0.01\cdot 340 \approx 3$ W/m$^2$.

**5.18** (a) It was stated earlier on that the incident average radiation intensity falling onto the surface of the rotating Earth was a quarter of the solar constant. The reflected intensity is thus

$$\frac{\alpha S}{4}=\frac{0{,}30\cdot 1380}{4}=100\,\frac{\mathrm{W}}{\mathrm{m}^2}.$$

(b) The average intensity hitting the atmosphere $\frac{S}{4}$, of which, reflected: $\frac{\alpha S}{4}$.

The average intensity absorbed by the Earth surface is the difference between the two: $\frac{S}{4}-\frac{\alpha S}{4}=\frac{S(1-\alpha)}{4}$.

$$\frac{S(1-\alpha)}{4}=\frac{1380\cdot 0{,}70}{4}=242\,\frac{\mathrm{W}}{\mathrm{m}^2}$$

(c) The released intensity (assuming a balance) is the same.

**5.19** (a) The instensity of solar radiation in a distance of $r$ (solar constant) is $S=\frac{L}{4\pi r^2}$

The radiation is received on a $R^2\pi$ cross section, but due to the rotation it will hit the $4R^2\pi$ surface of the planet, therefore the average incident intensity on the surface will be a quarter of the solar constant:

$$I_{\mathrm{be}}=\frac{S}{4}=\frac{L}{16\pi r^2}$$

(b) Replacing the orbital radiuses:

Mercury : 2310 W/m$^2$.
Venus: 663 W/m$^2$.
Earth : 345 W/m$^2$.
Mars: 149 W/m$^2$.

(c) If $\alpha$ is the planetary albedo, absorbed intensity will be $(1-\alpha)\frac{S}{4}$

In case of an equilibrium the absorbed intensity equals the emitted intensity. The planet is seen as a black body with a temperature of $T$:

$$I_{\mathrm{ki}}=\sigma\cdot T^4$$

$$(1-\alpha)\frac{S}{4}=\sigma\cdot T^4$$

$$T=\sqrt[4]{\frac{(1-\alpha)S}{4\sigma}}$$

Calculated temperature and albedo of planets:

| | | |
|---|---|---|
| Mercury : | 0.068 | 441 K (168°C) |
| Venus: | 0.770 | 228 K (−45°C) |
| Earth : | 0.306 | 255 K (−18°C) |
| Mars: | 0.250 | 211 K (−62°C) |

(d) For Mercury and Mars real temperature was close, for Earth and Venus the calculated value is a lot less.

The reason for this is that the calculated value is the temperature, at which the planet behaves like a black body and radiates towards the Universe. The model does not consider the substantial atmosphere Earth and Venus have, and the atmosphere does not emanate only outwards, but towards the surface just as well, therefore the intensity absorbed and emanated by the surface will be greater than that calculated here (this is the so called greenhouse effect.)

# Literature

W. J. Kaufmann: *Universe*, W. H. Freeman and Co., New York, 1988.

H. Karttunen, P. Kröger, H. Oja, M. Poutanen, K. J. Donner (Szerk.): *Fundamental Astronomy*, Springer, Berlin Heidelberg, 2007.

M. Seeds, J. Holzinger: *Student Observation Guide with Laboratory Exercises*, Prentice Hall, Englewood Cliffs, NJ, 1990.

T. L. Smith, M. D. Reynolds, J. S. Huebner: *Basic Astronomy Labs*, University of North Florida, 1996.

*ASTR 1010 Laboratory Manual*, *Introduction to Astronomy*, Dept. of Astrophysical and Planetary Sciences, University of Colorado, Boulder, 2016.

Bécsy B., Dálya G.: *The Nemzetközi Csillagászati and Asztrofizikai Diákolimpia Szakkör feladatai*: http://becsybence.web.elte.hu

http://www.stellarium.org
soho_realtime_hmi_Continuum
http://astronomie-smartsmur.over-blog.com
https://www.projectpluto.com/jevent.htmhttp://www.eso.org
http://www.csillagasz.at/
https://nssdc.gsfc.nasa.gov
http://spacemath.gsfc.nasa.gov
https://sites.google.com/a/uw.edu/introductory-astronomy-clearinghouse/assignments/labs-exercises
https://www.nasa.gov/mission_pages/tdm/solarsail/index.html
https://phys.libretexts.org/TextBooks_and_TextMaps/University_Physics/
https://courses.lumenlearning.com/astronomy/