Chaos Physics in Secondary School

A material applicable in online teaching

Ildikó Bajkó[1]

[1]Szent István High School Budapest, 1146 Budapest, Ajtósi Dürer 15.

Motto"But if you stir backward, the jam will not come together again."

Tom Stoppard: Arcadia

Abstract

Chaotic systems are not only subject for researchers, but we all encounter them in our everyday life, when, e.g., mixing cream in coffee. This paper addresses the issues of teaching chaos physics in high school within both extracurricular framework, and by incorporating it into high school physics curriculum. The teaching material is based on mechanical processes and has been tested in different student groups. The module was specially elaborated to conceptualize some basic aspects of chaos theory, like predictability, chaos, complex and chaotic motions, irreversibility, determinism and mixing. The post-tests, administered to students after completing the module, have shown significantly higher scores on conceptual questions, as compared to the results of the pre-tests, indicating a deeper understanding of the enumerated concepts. The curriculum is suitable for online teaching, too.

Keywords: chaos physics, high school

Introduction

Chaotic processes can be experienced in almost every branch of science. They are present not only in physical sciences, but also in many other disciplines, ranging from populations dynamics, via chemical reactions, to cardiac fluctuations. Many books available in the chaos literature give an overview of several chaotic systems (Gleick, 1987, Lorenz, 1993, Tél & Gruiz, 2006, Argyris et al., 2015), some others concentrate only on specific fields like, e.g. astronomy (Diacu & Holmes, 1996) or oceanic plankton patterns (Neufeld et al., 2003). Books devoted to artistic aspects of chaos have also been published (Sprott & Chapman, 2005), (Sprott, 2019), and basic chaos-related ideas appear even in theatre pieces, such as in Tom Stoppard's Arcadia (Stoppard, 1993). The choice is thus broad, for conceptual clarity, we concentrate on mechanical phenomena, mostly based on materials treated on undergraduate level in the introductory textbook (Tél & Gruiz, 2006). Because of the nature of chaos however, one can easily connect our examples to other scientific disciplines such as meteorology, astronomy, environmental issues, and even to arts: literature, handicrafts, and visual arts, via the phenomenon of mixing and irreversibility. Our experience shows that the essence of chaos physics can successfully be taught to students of different age, in teaching modules of different lengths.

The method

I am reporting about a teaching module I run in the last couple of years for a total of 72 students in their penultimate or last year of high school, in groups of 10-15 on average. These students came from classes specialized in science or math. Most teaching activity took place within extracurricular framework in study groups, but the program has also been incorporated into the standard curriculum by devoting a few hours to it in the Mechanics or Modern Physics chapters. In this latter case, the teaching material was reduced to a 4-5 hours frame. The teaching material I prepared can also be offered as a project work for students, then it is possible to delve deeper into various topics. It worked perfectly at our school's yearly week-long project-based teaching. Moreover, the teaching unit has successfully been applied for the purpose of distance learning in the spring semester 2020 during the online teaching period in Hungary.

Before students heard anything from us about chaotic phenomena, they wrote a pre-test. In the very first lesson of the module, the students become acquainted with simple physical systems that exhibit chaotic behavior: magnetic pendulum, driven pendulum, elastic pendulum, ball bouncing on double slopes. We present here very briefly the devices used by us, most of which can be easily assembled with the tools available in a physics lab. Their detailed description can be found in (Tél & Gruiz, 2006). Students had time to make the experiments themselves.

Magnetic pendulum

Consider a pendulum, the end-point of which is a small magnetic body, moving above three identical magnets placed for example at the vertices of a horizontal equilateral triangle.[i] When the forces between the pendulum bob and the magnets are attracting, the pendulum can come to a halt, pointing towards any of the three magnets. Thus, there are three simple attractors in the system.

The driven pendulum

We examine the case when the suspension point of a pendulum is moved sinusoidally in time along a horizontal line. The pendulum bob is fixed to a very light, thin rod, or we can use a ruler as a physical pendulum. With a sufficiently strong driving, the motion may become chaotic.

Elastic pendulum

Consider a pendulum having its string replaced by a spring of rest length l_0 and natural frequency ω_0. This can also serve as a model of a ball attached to an elastic cord, called shimmy ball. In contrast to a real shimmy ball, we assume that the cord never loosens.

Ball bouncing between double slopes

A ball is bouncing on two slopes of identical inclination facing each other in a gravitational field. Chaotic behavior arises because after bouncing back from the opposite slope the ball does not necessarily hit its original position. Non-linearity and inherent instability are caused by the breakpoint between the slopes. During this experiment, students video-record the position of the ball as a function of time and digitalize it with the *Tracker* program. By comparing two time series of nearly identical initial conditions, they are able to witness butterfly effect in their own experiment.

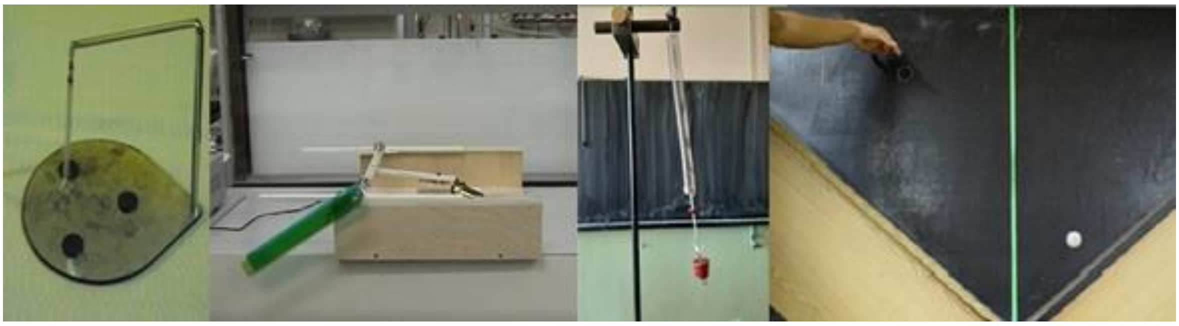

Figure 1 Simple chaotic tools: magnetic pendulum, driven pendulum, elastic pendulum, and ball bouncing on double slope used in the introductory, experimental part of the teaching module (photos taken by the author)

The very important take-home message from this part of the course was that even simple systems can behave unexpectedly in a complex way, a property that can also be used as a definition of chaos (Tél & Gruiz., 2006).

In the subsequent classes/lessons we complemented the study of these simple systems with numerical simulations, using an in-house program written by my colleagues. We also showed that Excel can easily be used to monitor motions in time. For advanced students with special interest in informatics the use of Dynamic Solver was suggested (Csernovszky et al., 2020).

In parallel with the numerical simulations, we introduced also the concept of the phase space. Phase space refers to the plotting of both the particle's velocity **(v)** and its position **(x)** on a two dimensional graph (Figure 2.). One given state of the particle (i.e. pendulum bob or ball in our case) is represented by a single point in the phase space, while the *movement* of the particle by a series of subsequent points. In other words, any real-life movement traces out a path in the phase space that is called trajectory. It is important to emphasize that real-life trajectories (i.e. those that we see in the physical space) are different from the phase space trajectories, so they might have very different shapes. Time does not appear explicitly in the phase space: it is represented solely by arrows pointing along the phase space trajectories to indicate in which direction the given trajectory evolves in time.

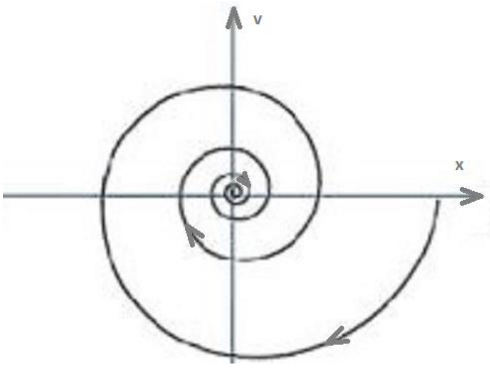

Figure 2 Example of a (non-chaotic) phase space trajectory showing the possible values of the variable pair: position and velocity, along the motion. (schematic drawing of the author) This phase space trajectory corresponds to a damped oscillation that finally stops at x=0.

Time series belonging to chaotic systems do not exhibit any regularity over arbitrarily long stretches of time. To visualize them, a phase space representation is particularly useful since it has a finite extension, independent of time. Figure 3 presents such a chaotic trajectory. The lack of any repetition is clear from the short piece shown here (the trajectory of full length would shade an area of the plane).

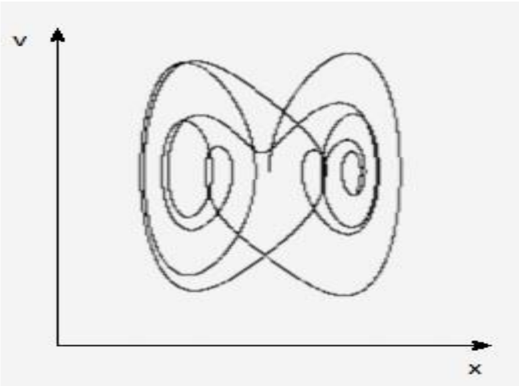

Figure 3 An example of a chaotic phase space trajectory, which is complicated, and resembles an irregular coil (result of a numerical simulation of the author)

It is surprising to see that applying a properly chosen sampling procedure to a long chaotic phase space trajectory, an unusual but well-structured pattern appears. The scientific term for such an object is chaotic attractor. An example is shown in figure 4.

Figure 4 Pattern of a properly sampled long chaotic trajectory, piece of a chaotic attractor (result from a numerical simulation of the author)

The pattern illustrates clearly that chaos is by no mean a complete disorder, it exhibits a strict geometric structure when it is appropriately observed. This is a consequence of the deterministic nature of the dynamics, of Newton's law in mechanical examples. Such complex patterns are examples of fractals (Mandelbrot, 1982). Students may discover that in the case of chaotic systems, a fractal structure always appears in the phase space.

Via the experiments and computer simulations, students can formulate the three most important characteristics of chaos (Tél & Gruiz, 2006): (i) irregularity of the motion; (ii) unpredictability, i.e. sensitivity to initial conditions; (iii) order, specific geometric pattern, i.e. the appearance of a fractal structure in the phase space. This step-by-step discovery process aroused a great enthusiasm among the students.

We have also discussed the known forms of motions, and discovered that the regular periodic motion of a pendulum becomes irregular immediately when placed between the poles of a few magnets. The Earth's orbit around the Sun is periodic, but Pluto's orbit is already chaotic (Diacu & Holmes, 1996).

The world of fractals in itself enchanted the students**,** thanks also to the beautiful seahorses, whirls, islands and gulfs that appear in Mandelbrot and Julia sets (Mandelbrot, 1982). It is a gratifying feeling that forms that occur so frequently in nature as e.g., trees or leaves can be described by the language of science (Vicsek, 1992). Students were surprised by the fact that mountains and clouds can be generated by computers as fractals. We discussed, however, that the vast majority of fractals is not related to chaos.

Basic concepts

In connection with chaos it is natural to discuss the question of predictability in high schools (Tél & Gruiz, 2006). Egyptians knew the exact minute of the solar eclipse, but to forecast the weather precisely is a challenge even today. Not only complex phenomena such as weather forecasting can be problematic, predicting the motion of a magnetic pendulum, or of any chaotic system, is difficult, too.

When experimenting with a magnetic pendulum during the teaching module, we always follow the trajectory of the pendulum two times in succession, starting from a rest state at the same position, as much as possible. Already after a short time, a small difference in the two trajectories can be seen, and after a while the two trajectories become completely different. Examining the trajectories of a driven pendulum, we observe similar features. Some of the students were able to explain this by pointing out that the two starting points were not exactly the same. They also added that in their views, this does not justify such a large difference after a few seconds. Therefore, we verified our experimental observation by computer simulations. We examined the path of a driven pendulum on a computer. The program was run twice with two different starting conditions differing in the fifth decimal digit. We printed out the paths and compared them. The simulation provided quickly divergent orbits, although here we knew exactly how small the difference between the two initial conditions was. It has been proved that, despite knowing the equations, the motion of the driven pendulum is unpredictable.

The term butterfly effect was mentioned as a synonym of sensitivity to initial conditions, which has gained world fame due to Gleick's book written to popularize chaos (Gleick, 1987). The students agreed that a huge risk of misunderstanding lies in the use of this term, which might suggest a kind of nihilism, when not stating that there are well predictable statements even within chaos (e.g. that the attractor will be reached for sure, and the statistical features of the motion on it can fully be predicted).

The purpose of the last lesson was to synthesize and summarize. In this last hour of the teaching program often a heated discussion arose: if we know the equations of motion, why is it not possible to predict the system's behavior. I was faced in all cases with a very stable deterministic worldview from the part of the students. In order to weaken it, we have discussed that the nonlinear term (like a quadratic term or sine term) in the system-describing equations of motion is the one that appears to magnify the initially small errors and uncertainties. While for linear equations the error increases at most linearly in time, here it grows exponentially. This is what gives rise to the sensitivity to initial conditions. Then the next question came: why do we talk about chaos – which in its everyday use means disorder - if we know the equations? There was only one possible answer to this: we must call this phenomenon deterministic chaos. It is exactly the deterministic attributive which tells us, that this is different from disorder. They accepted this explanation. One group even formulated that it was not to promote understanding that this phenomenon was named chaos.

The teaching module and its two versions

Table 1 shows how our teaching module is structured. Before and after the teaching program, students completed a test so that we could follow the changes in their understanding. In the last session we had a free discussion about the experiences and findings.

Table 1 Curriculum for the Chaos Physics teaching module, for study groups

Lesson number	Theme	Content, concepts
1.	Filling pre-test	
2	Introduction. Simple mechanical systems with chaotic behavior	Magnetic pendulum. Driven pendulum. Elastic pendulum. Double slope
3-4.	Examples of chaotic motions - Computer simulations	Computer simulations: - motion in real space - parameter settings - sensitivity to initial conditions - phase space - chaotic attractor
5.	Fractals	Fractals in Nature. Mathematical fractals (Cantor set, Cantor filaments, etc.)
6-7.	The Concept of Chaos through experiments: Pendula, bouncing between double slopes	Periodic - Chaotic Motions Linearity - Nonlinearity Predictability - Sensitivity to initial conditions Deterministic Chaos
8.	Is chaos useful?	When is chaos useful? – Mixing Irreversibility Stability-Instability
9.	Repetition. Summary	
10.	Filling a Post-test	
11.	Synthesizing discussion	

As mentioned before, I also incorporated the program into the standard curriculum, by devoting a few hours to it. To this end, the teaching material was reduced to a 4-5 hours unit. I taught five times in the class, in different school years, for groups of 20-26 students coming from the same age-groups. This was done in the framework of Mechanics, following the chapter Vibrations, Waves. I have told the students that they are essentially learning a new chapter of Modern Physics. We have not entered deeply into simulations here; we have only looked at the characteristics of chaos in connection with simulations taken from the internet (see e.g. endnote ii). In this version, one has the opportunity to use results obtained by students of earlier study groups, like e.g. their Tracker digitized time series, or video-recorded experiments. I also taught -successfully- this version of the chaos physics curriculum in online teaching with few modifications, in the form of e-learning material.

Development of chaos concepts in view of the responses to tests

One recurrent query in the tests is related to the experiments carried out with magnetic pendulum.[ii]

What kind of motion will the magnetic pendulum exhibit in the figure below, if it is started with a very small deviation from the middle? (In our set-up the magnets on the table repel the pendulum.) Describe the motion or mark it in the figure! (Figure 5)

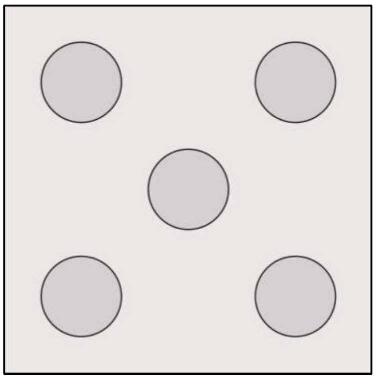

Figure 5 Image in the test to draw the trajectory of the magnetic pendulum

Answers to this question are summarized in the following table (this is the average of the tests filled in so far):

	regular motion	irregular (chaotic) motion	Comment
pre-test	67%	33%	
post-test	10%	90%	47% - chaotic motion 33% - irregular motion 10% - sensitive to initial conditions

After completing the teaching module including the experiments, it was accepted by the students of all the different groups that even the motion of systems with a few components can be complicated.

One question from the questionnaire was to find out the interpretation of the concept of chaos

What do you mean by chaos? Explain briefly! ....

All the students defined chaos in the pre-test according to the everyday vocabulary, as disorder, messy phenomenon, etc. Very few of the students also referred to unpredictability. In the post-test, only about 17% of the respondents identified the concept of chaos exclusively as disorder and confusion. The others have changed their interpretation: 80 % of the responses included unpredictability: from these there were 23% exact definitions, and there were 13 % in which disorder and unpredictability was mentioned together.

The concept of chaos was in focus of another question, too:

Which of the following systems' motion would you call chaotic (multiple answers are allowed):
a. a very messy room
b. a poorly organized running race
c. mixing of paints

d. weather

The answers to this question are summarized in the following table.

	a	b	c	d
pre-test	50%	53%	33%	33%
post-test	20%	27%	90%	60%

From these answers, there is a clear shift from the everyday concept of chaos to the scientific one. It also reveals that, in very few cases, the two concepts coexist. Thus, almost every student has become acquainted with the concept of deterministic chaos.

Focus on mixing and irreversibility

Our teaching material pays special attention to the mixing of liquids because it is in this phenomenon where the appearance of order, a fine geometric structure, can best be captured. Fractal patterns typically occur in chaotic processes in an abstract mathematical space, the phase space, and are thus hidden from direct observation. Mixing is the exception, students were told that in this case the phase space coincides with the real space, so chaos-related fractality can be observed in everyday life: milk or cream mixed into coffee, raspberry syrup poured into milk custard, dirt or stains on the surface of lakes or rivers. In all these cases filamentary structures appear.

As an introductory remark, we show students that folding and stretching two or three different colored plasticine rods produce a similarly spectacular pattern (Figure 6).

Figure 6 Filamentary pattern on a plasticine stick of two colors (photo taken by the author)

It also became clear that in relation to dough kneading our grandmothers have long known what science recently formulates: best mixing is provided by the stretching and folding algorithm. Mixing is most effective when the process is chaotic: so by this example we can discuss one of the useful applications of chaos.

High school students were delighted to create fractal structures on their own. We showed them the "marbling" technique on paper and candle (Szatmáry-Bajkó, 2016). The beautiful patterns of filamentary fractal structure enchanted them (Figure 7).

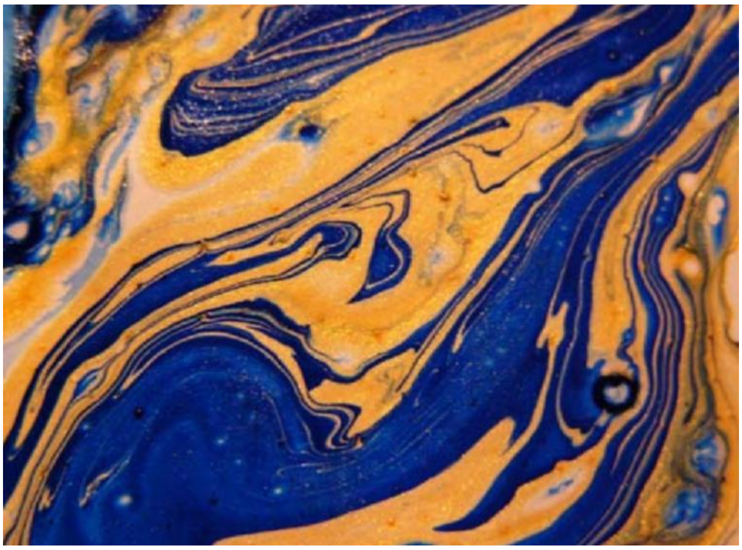

Figure 7 Patterns of filamentary fractals obtained by marbling technique (photo taken by the author)

The girls were especially enthusiastic when it became clear to them that in addition to a sheet of paper and candles, even aesthetic scarfs can be obtained with this technique. Here again, we had the opportunity to turn back to the patterns occurring in nature: the filamentary patterns resulting from this technique evokes the sight of marble.

Structures similar to those resulting in marbling activities appear in our environment, for example in the case of the spread of contaminants. Students have the possibility to compare the pattern of their handicraft obtained by marbling and of oil contamination on the surface of water or of foam pollutants before a dam. They thus learn about an important aspect of environmental science: the spreading of pollutants is a chaotic process. The related pattern is not a simple compact patch, rather it always exhibits fractal-like filaments.

I always noticed a smile on the face of the students when they turned to the question "mixing jam in rice pudding". This question touched the notion of irreversibility. Before this question was asked in the post test, during the teaching material we always discuss a dialogue from Tom Stoppard's play Arcadia (Stoppard, 1993):

"*Thomasina: When you stir your rice pudding, Septimus, the spoonful of jam spreads itself round making red trails like the picture of a meteor in my astronomical atlas. But if you stir backward, the jam will not come together again. Indeed, the pudding does not notice and continues to turn pink just as before. Do you think this is odd?*
Septimus: No.
Thomasina: Well, I do. You cannot stir things apart."

The full question on this topic was:

Slowly, with a few moves, we mix raspberry jam in rice pudding.

1. If you slowly stir with the same movements backwards, will the raspberry jam return to its original position: a. yes b. no.

2. Is this motion predictable?

The answers are summarized in the following table:

	yes	no	comment

Return to original location?	0	100%	
Is this motion predictable?	57%	43%	One student, who responded no, explained: it would be predictable if the circumstances were perfectly accurate

In summary, the answers show that it is clear for all students that the process is irreversible, even if some of them were not familiar with the scientific meaning of the concept. A related question was also asked on this topic:

Slowly, in a few steps, we mix the raspberry jam into the rice pudding. If you mix it slowly with the same movements backwards, we know that the raspberry jam will not return to its original position. With this in mind, do you think that chaos has anything to do with the fact that in reality, processes always go in a particular direction (irreversibility)?

The answers to this open question were very scattered, with a total of 77% thinking that chaos had to do with irreversibility. I would highlight an answer that demonstrates understanding: “Theoretically, if you make the same movement backwards on the same path, the process can be reversed. But this is not possible in practice.”

We intended to capture the students’ understanding of the concept of determinism with the following question:

Which of the following do you think is more characteristic to our world of physical phenomena?

a. a world where laws determines exactly what will happen later, or

b. a world where the existing uncertainties grow, and so probabilistic concepts play an important role.

Again, the answers are summarized in a table:

	deterministic worldview	acceptance of sensitivity to small uncertainties	both options accepted
pre-test	83%	17%	0
post-test	50%	40%	10%

On the one hand, we can see an increase of the acceptance of sensitivity to small deviations, in fact the increase in the acceptance of the role of chance. On the other hand, the coexistence of the two worldviews characterizes 10 % of respondents.

Summary

Our experience shows that chaos is exciting even for high school students. The visual and formal possibilities of chaos attract students' attention, help deepen scientific thinking, and awake their creativity. Although we are aware of the challenges the teaching of these phenomena at secondary school level imply, because e.g. the appropriate mathematical apparatus can’t be used, we argue that it is very important for students to encounter this subject during their school years, because otherwise the oversimplification of nature’s description can deprive students’ belief and trust in the power of science. Further advantages of the material are: direct connection with multimedia methods (Tracker, Excel, Dynamics Solver), interdisciplinarity (meteorology, astronomy, environmental aspects), and relation to arts (theater piece, handicrafts, painting). These features characterize the material even if applied in online teaching. Therefore, we find worth looking ahead and to cooperate with the teacher community in this matter.

Acknowledgements: Useful discussions with I. Benczik and T. Tél are acknowledged. The project is funded by the Content Pedagogy Research Program of the Hungarian Academy of Sciences.

References

Argyris, J., Faust, G., Haase, M., Friedrich, R. (2015). *An exploration of Dynamical Systems and Chaos*, Springer, New York.

Csernovszky, Z., Nagy, P., Tasnádi, P., (2020). Investigation of chaos in the absence of programming skill, *Canadian Journal of Physics, 98, 593-605.*

Diacu, F., Holmes, Ph. (1996). *Celestial encounters: The Origins of Chaos and Stability*, Princeton Science Library.

Gleick, J. (1987). *Chaos: The Making of a New Science*, Viking Books.

Lorenz, E.N. (1993). *The Essence of Chaos*, University of Washington Press.

Mandelbrot, B. (1982). *The Fractal Geometry of Nature*, H. Freeman and Co.

Neufeld, Z., Hernandez-Garcia, E. (2003). *Chemical and biological processes in fluid flows: A dynamical systems approach*, Imperial College Press, London.

Sprott, J. C., Chapman, R. (2005). *Images of a Complex World: The Art and Poetry of Chaos*, World Scientific, Singapore.

Sprott, J.C. (2019). *Elegant Fractals: Automated Generation of Computer Art*, World Scientific, Singapore.

Stoppard, T. (1993). *Arcadia*, Faber & Faber.

Szatmáry-Bajkó, I. (2016). Handicraft and aesthetic experience in teaching chaos physics, in: A. Király, T. Tél eds., *Teaching Physics Innovatively*, pp. 15-20**.** ELTE Budapest.

Tél, T., Gruiz, M. (2006). *Chaotic Dynamics, An introduction based on classical mechanics*, Cambridge University Press, Cambridge.

Vicsek, T. (1992). *Fractal Growth Phenomena*, World Scientific, Singapore.

---

[i] One version of the magnetic pendulum can also be ordered online: ROMP (Random Oscillating Magnetic Pendulum), for example at http://www2.oberlin.edu/physics/catalog/demonstrations/waves/romp.html

[ii] It is worth also discussing with the students this video available on the internet: Magnetic pendulum – Dynamic Geomag https://www.youtube.com/watch?v=Qe5Enm96MFQ