A Simulation Experiment Using Algodoo: What Force Makes a Car Accelerate, and What Does the Acceleration Depend On?

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In a physics lesson when you ask the question "What force makes a car accelerate?" the most frequent answer you get from your students is "The force of the engine." Obviously, the role of the engine is important, but the acceleration is dependent on external factors, since an object cannot exert a force on itself to change its own momentum. Even a car with a very powerful engine will be unable to start on ice or in deep mud or sand. On the other hand, a weak engine is unable to make a heavy car move. In this paper, we present a simulation-backed approach supporting the teaching of this topic in secondary school. The simulations were created in the free program Algodoo.

In recent times, the secondary school physics syllabus in Hungary has shifted toward applications of science in our everyday life. As a part of this transition, main topics ended up having everyday titles like "The physics of modes of transport on land, water, and air" or "Physics of sports activities" rather than theoretical titles such as "Kinematics" or "Dynamics." The motivation behind choosing these topic titles is clear: they show students that everything surrounding them in the world can be modeled using the laws of physics. A clear benefit of this approach is that it creates extra motivation for students interested in transportation or sport. In addition to this, the physics of transportation provides an opportunity to underline the importance of science: human lives may depend on cars slipping, overturning, or being unable to stop before an obstacle. Studies of these phenomena appear in literature, for example, wheel slipping when starting a car,[1] tailgating accidents,[2] the role of road inclination angle in accidents,[3] and rollover of cars.[4]

The difficulty of this application-based approach is that these topics are more difficult to experimentally support in classroom conditions than the topic of kinematics in general; this is simply because cars and planes do not fit into classrooms. However, we can still measure the distance covered and the time taken by a vehicle using a tape measure and stopwatch, from which the velocity and acceleration of the vehicle can be calculated.[5] In addition, the movement of vehicles can be analyzed using videos[6–9] and sensors located in cars,[10–13] helping our students to arrive at a physically correct model starting from their originally existing concepts.

The difficulty of the topic of dynamics is that while basic quantities of kinematics (distance, time, velocity, and acceleration) might be experimentally measured or analyzed using a video, the basic quantities of dynamics (force, linear momentum, and energy) cannot be directly measured or analyzed, especially in the case of a large vehicle. There are many examples in the literature about overcoming this problem for some phenomena: it is possible to calculate force using an acceleration sensor,[14] or to calculate power and torque acting on wheels by measuring the angular frequency of the wheel.[15]

In classroom conditions, we can model real traffic situations using small carts. This method enables us to investigate situations that would be dangerous in real life, such as the rollover of a car,[16] or instabilities during braking leading to the spinning of a car.[17]

These demonstration experiments and measurements are spectacular, and if parameters are correctly chosen, they give a good illustration of reality. However, for an accurate understanding of the physics behind it, students should know what forces act on the car. The real difficulty of these situations is that forces cannot be seen either in reality or in demonstration experiments. Therefore, the only way to find out the net force is to measure the acceleration, but this still does not provide any information about particular forces creating the net force. This difficulty might be overcome by using simulation experiments to complement the demonstration experiments.

Algodoo is a free simulation software that allows users to create their own digital experiments or modify ones created by others, then run the simulations and evaluate them by rerunning, stopping, or playing them in slow-motion mode. The biggest advantage of the program is that many experimental configurations can be modeled within mechanics; this diversity was presented by Gregorcic and Bodin[18] and Euler et al.[19,20] Çoban[21] showed the significance of the program in online education by giving an example of teaching the physics of collisions in Algodoo. Furthermore, the program can also handle gravity as a central force field; therefore, it is suitable for demonstrating and exploring Kepler's laws.[22] It also includes a fluid model; therefore, it can be used for simple demonstrations in hydrostatics as well.[23] The program is very intuitive and explained step by step on the program website,[24] so we do not discuss the technical details in our paper but highlight its role in physics education instead.

The aim of this study is to discuss and explain the complex problem of starting a car by discussing simple cases step by step in detail. As the theoretical knowledge of the students is not deep enough, motion simulation experiments can be helpful.

Results and their evaluations

At the beginning of the topic, it is worth asking the class the question, "What force makes a car accelerate?" There are many possible answers to this. Of course, based on everyday knowledge, students tend to answer "the accelerating force of the engine," but this is too vague and not easy to fit into our

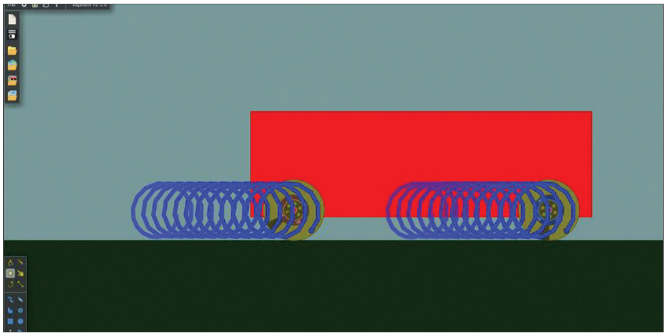

Fig. 1. A simple model of a car starting on an icy road in Algodoo. The wheels are sliding as shown by the tracker placed on the edge of the wheels.

knowledge about forces in physics.

By emphasizing Newton's laws and asking the right questions, we can lead the students to realize that there is a definite need for interaction with an external object in order to accelerate. The car is essentially in contact with its surroundings only through its wheels, so the accelerating force must surely act there.

The engine makes the wheels rotate, but in order to start moving in the horizontal direction, the car must interact with an external body, and assuming that the car is on a level surface, the only force that can act horizontally on the car is the friction between its wheels and the ground. Thus, the only force that can accelerate a car is friction. From this line of thought, it is easy to understand why even the strongest sports car will be unable to start on icy roads.

This phenomenon can be illustrated by a hands-on model experiment by placing a remote-controlled car on a tray of ice in its melting phase (the surface of ice should be slightly wet). When the car is started, the engine rotates the wheels, but the car hardly moves (it remains roughly stationary), since the wheels are sliding on the surface, causing only a very small acceleration. Our simulation experiment series also starts with investigating the case of sliding wheels.

The initial step is to create a simple car model in Algodoo, as shown in Fig. 1. The parameters of our setup are the following: the total mass of the car is 1220 kg (out of which 10 kg is the mass of each wheel), the radius of the wheels is 40 cm, the frequency of the motor driving both the front and back wheels (four-wheel drive) is 300 rpm (i.e., 5 rotations/s), and the torque of the engine is 500 N · m. Our first simulation investigates the start of a car on a slippery, icy road, so the friction parameter should be set to 0.1 for both the wheels and the ground.

At the start of the simulation, the wheels will be in sliding rotation, and it takes a long time for the car to reach its maximum velocity, which can be calculated by multiplying the frequency of rotation by the circumference of the wheel, which gives $v = 12.6$ m/s. The motion can be further investigated by placing a tracker onto the circumference of one of the wheels.

The velocity–time and acceleration–time graphs of the model car are shown in Fig. 2. The car accelerates until a force of friction is acting on the sliding wheels. After reaching the maximum velocity, the wheels will undergo rolling without slipping. The accelerating phase can clearly be seen on the graphs, and the value of the acceleration ($a = 0.98$ m/s^2) can be read from either the velocity–time graph as the slope or the acceleration–time graph as the constant value in the first phase.

The value of acceleration (read from the graphs) can be easily checked by a simple calculation. The total friction (F_f) between the wheels and the road is calculated as the product of the coefficient of friction ($\mu = 0.1$) and the normal force (mg) acting on the wheels:

$$F_f = \mu mg \approx 1196 \text{ N}. \quad (1)$$

The acceleration then can be found using Newton's second law:

$$\alpha = \frac{F_f}{m} = \mu \cdot g = 0.98 \frac{\text{m}}{\text{s}^2}; \quad (2)$$

thus, the value of acceleration obtained from the graphs of the simulation and the result of our theoretical calculation agree. Note that the program does not distinguish between the coefficients of static and kinetic friction. Of course, in reality, this is not true. Therefore, it should be emphasized that starting with spinning wheels results in less acceleration because the coefficient of kinetic friction is less than the coefficient of static friction.

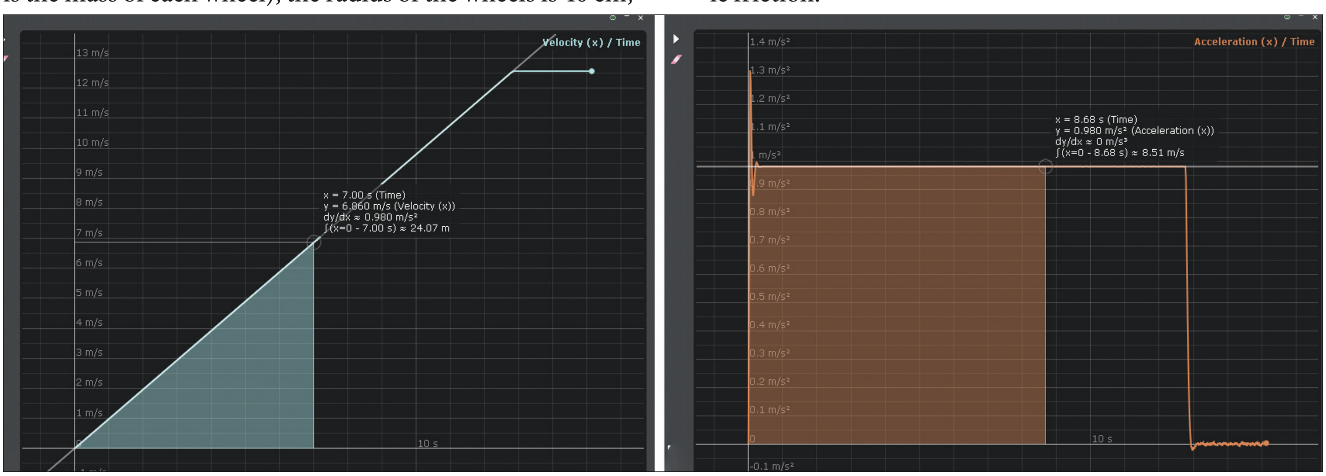

Fig. 2. The velocity–time and acceleration–time graphs of a car starting on an icy road in Algodoo.

Numerous versions of the basic simulation can be carried out by changing the parameters in our original setup. Since these extra tasks can be easily carried out by modifying our original simulation, these can be given out to the students in the form of homework or project work. In addition to carrying out the modified simulations and evaluating the results, students should also be encouraged to find a physical explanation of their results.

• *Does the frequency of the engine have any effect on the acceleration?*

Let us double the original frequency of both engines. As a result, the maximum speed of the car doubles, but the time taken to reach that speed also increases to twice the original, leaving the acceleration unchanged. If the original frequency is halved, the maximum velocity and the time taken are both halved; thus, the acceleration remains the same. This result can be justified by the fact that friction force does not depend on the relative velocity of the two contact surfaces.

• *How does the acceleration of the car change if we accelerate only its rear wheel or only its front wheel (cases of rear-wheel- and front-wheel-drive cars)?*

In order to check this, we first replace the front motor with a simple frictionless axle and thus create the model of a rear-wheel-drive car. If the simulation is run with this new setup, we can observe that the acceleration is clearly less than the acceleration in the case of the four-wheel-drive model. This is because now the only force accelerating the car is the friction acting at the rear wheel, which is approximately half of the friction acting in the case of the four-wheel-drive car. The friction force is halved because the simulated car is homogenous and symmetrical, so approximately half of the normal force acts on the rear wheel. It is important to note that in real life, the normal forces acting on the front and rear wheels usually differ. The heaviest part of the car is the engine; hence, placing it in the front of the car works better with front-wheel drive due to the ratio of the normal forces, and also, this way, power transfer to the wheels requires less parts.

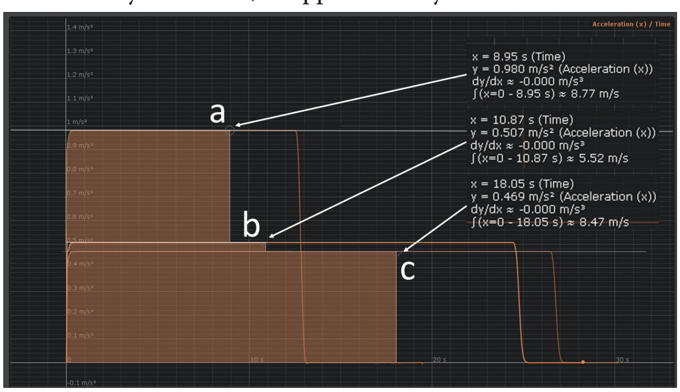

Fig. 3. The acceleration–time graphs of four-wheel-drive (a), rear-wheel-drive (b), and front-wheel-drive (c) cars.

Let us change the simulation so that only the front wheel is driven by a motor. In this case, the acceleration decreases again, but interestingly, its acceleration is slightly less than in the case of the rear-wheel-drive car (see Fig. 3).

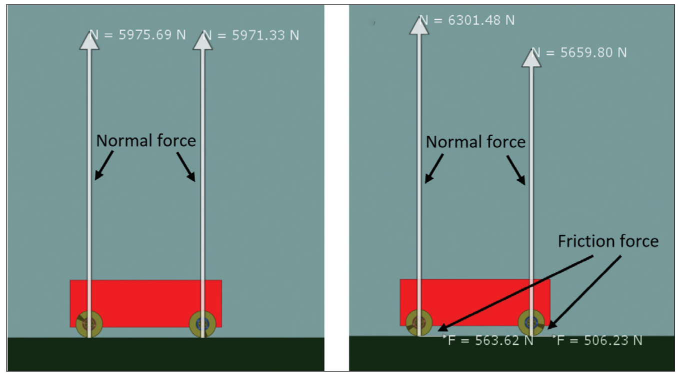

Fig. 4. The visualization of the normal forces and frictions acting on the wheels of a car moving with constant velocity (a) and an accelerating car (b) in Algodoo.

At this point, we can discuss the temporary change in the ratio of the two normal forces acting on the wheels when a car starts. It can be illustrated in Algodoo by turning on the visualization of the forces. Figure 4 shows the normal forces and frictions acting on the wheels, and it can be seen that the normal forces acting on the front and rear wheels are similar when the car is not accelerating (a), and they are different when the car is accelerating (b).

In the case of a rear-wheel-drive car with a powerful engine, high torque together with high friction causes the normal force acting on the front wheels to decrease significantly; in extreme cases, this might result in the front of the car rising into the air. A simulation can be used for a convincing demonstration of this phenomenon. In order to achieve the desired effect, we change the torque of the motor of the rear-wheel-drive car to the maximum value and set the coefficient of friction of the ground to infinity. If the simulation is run with this setup, the front of the car lifts from the ground at the start. It should be noted that it can be calculated theoretically that while a car is accelerating, the net torque acting on the whole car about its center of mass can be 0 only if the normal forces acting on the wheels are different. This is true because if they were equal, then their torque would cancel each other; therefore, there would be no force whose torque can balance the torque of friction.

• *Does changing the motor torque affect the acceleration of the car?*

Let us change the torque of the engines to twice ($1000\ \mathrm{N \cdot m}$), then four times ($2000\ \mathrm{N \cdot m}$), the torque in our original simulation. Algodoo shows that the simulated car accelerates exactly the same as before, so we can conclude that on slippery roads, increasing the torque does not increase the acceleration of the car. It is worth noting here that Algodoo graphing has a very helpful feature for comparing two accelerations with different torque values by keeping the graph of the previous simulation and drawing the graph of the new one onto it, thereby facilitating the comparison of results. Let us change the torque of the engines to $100\ \mathrm{N \cdot m}$. If we run the simulation now, the car's acceleration is noticeably smaller. To explain this, it should be considered that the maximum value

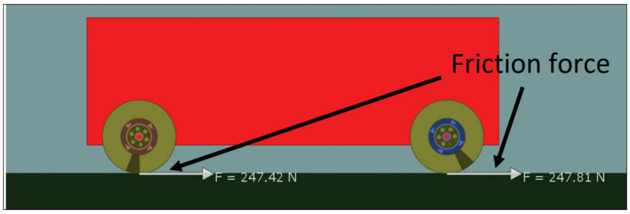

Fig. 5. Friction acting at the wheels in the case of a low-torque motor. In the non-slipping regime, the friction force (and thus the acceleration of the car) is proportional to the torque of the motor.

of static friction is determined by the coefficient of friction and the normal force; therefore, if the torque of the motor is small enough, this maximum friction might not be produced at the wheels.

In case of the wheels accelerating without slipping, the torque of friction (F_f) acting on a wheel about its axis should be less than the engine torque on a wheel (τ). The difference between the two torques accelerates the rotation of the wheels:

$$\tau - F_f \cdot R = I \cdot \alpha, \tag{3}$$

where R is the radius of the wheel, I is the moment of inertia of the wheels, and α is the angular acceleration of the wheel. In the simulation, we modeled the wheels with cylinders, so we used

$$I_{\text{cylinder}} = \frac{1}{2} m_w \cdot R^2, \tag{4}$$

where m_w is the mass of a wheel. The net force acting on the car is the sum of the two frictions acting on the wheels. In this case, because of the rolling without slipping, the angular velocities of the two wheels are always equal, so their angular accelerations are also equal, so it can be calculated from Eq. (3) that it can only happen if the same friction forces act on both wheels, as can be seen in Fig. 5. Applying Newton's second law on the car,

$$2F_f = m_c \cdot a, \tag{5}$$

where m_c is the mass of the car (including wheels), and a is the acceleration of the car. Since the car accelerates without its wheels slipping, the following equation can be written for the relationship between the acceleration of the car and the angular acceleration of a wheel:

$$a = \alpha \cdot R. \tag{6}$$

By solving the system of Eqs. (3), (5), and (6) for the three variables F_f, α, and a with the parameters $\tau = 100$ N · m, $R = 0.4$ m, $m_w = 10$ kg, and $m_c = 1220$ kg, the following solution can be obtained:

$$F_f = \frac{\tau}{R\left(1 + \frac{m_w}{m_c}\right)} = 248 \text{ N} \tag{7}$$

$$a = \frac{2\tau}{R\left(1 + \frac{m_w}{m_c}\right) m_c} = 0.4065 \ \frac{\text{m}}{\text{s}^2}, \tag{8}$$

where the acceleration is smaller than the maximum value calculated in Eq. (2). The magnitude of friction acting on the wheels can also be verified by making the program visualize the forces, as shown in Fig. 5.

It should be noted that because the weight of the wheel is much smaller than the weight of the car, a simplified equation can be derived for the acceleration:

$$a = \frac{2\tau}{R\left(1 + \frac{m_w}{m_c}\right) m_c} \approx \frac{2\tau}{R m_c}, \tag{9}$$

where the obtained value only differs by 0.8%, giving a useful estimation.

Cars start moving with no slippage of the wheels when their acceleration is limited by torque, not friction. In that case, the acceleration can be calculated from Eq. (8) or the simplified Eq. (9), while the maximum achievable acceleration is set by Eq. (2). Overall, we can conclude that a car will start without slipping if the following inequality is met:

$$\mu m_c g > \frac{2\tau}{R\left(1 + \frac{m_w}{m_c}\right)} \approx \frac{2\tau}{R}, \tag{10}$$

but if the driver increases the torque too much, so that inequality is no longer met, the wheels slip. Modern cars with a traction control system do not let the motor break this limit; the driver can feel that under certain conditions, the computer restricts the torque of the motor in order to prevent the slip.

• *Does changing the friction coefficient affect the acceleration of a car?*

It might be interesting to systematically investigate the effect of changing the friction coefficient on the acceleration of a car. It can be seen from Eq. (10) that for a given torque, there can be identified a threshold value of the coefficient of friction

Table I. The acceleration of the model car in Algodoo using different friction coefficients.

μ	0.01	0.02	0.03	0.04	0.041	0.043	0.045	0.05	0.06
a (m/s^2)	0.098	0.196	0.294	0.392	0.402	0.406	0.406	0.406	0.406

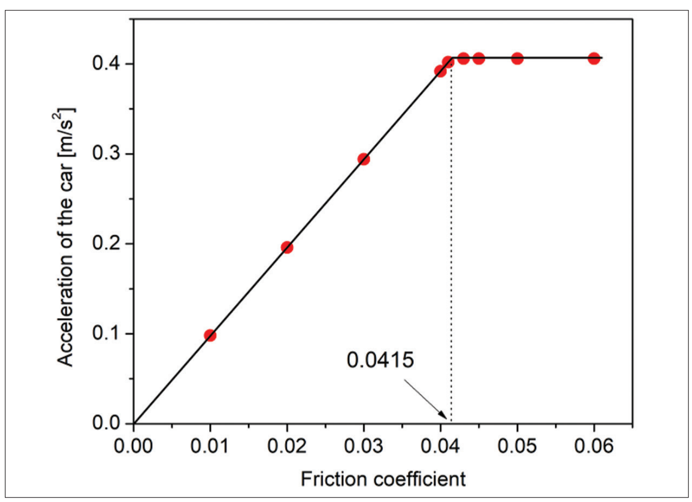

Fig. 6. The acceleration of the simulated car with motor torque kept constant as a function of the friction coefficient.

where wheels start slipping. This threshold can be calculated as follows:

$$\mu = \frac{2\tau}{m_c g R\left(1+\frac{m_w}{m_c}\right)} = 0.0415. \quad (11)$$

Below this value, the acceleration is directly proportional to the friction coefficient, while if $\mu > 0.0415$, the acceleration is limited by the torque of the motor. This means that the change of the coefficient does not affect acceleration, as can be seen in Table I and Fig. 6. It should be noted that if a desired coefficient of friction is to be achieved in Algodoo, the friction parameter of both surfaces should be set to the same number, which should be the same value as the desired coefficient of friction for the two involved surfaces.

Table I verifies the calculations for the threshold value of the friction coefficient and the acceleration that can be calculated from Eqs. (2) and (8).

In real life, cars can hit a puddle or oil patch on the road, so the friction coefficient temporarily lowers between the road and some of the wheels enough to make them slip. This phenomenon can lead to an accident, especially in a road bend. But even on a straight road, the difference between the friction forces acting on the wheels creates a torque about the vertical axis of the car, which can be large enough to spin the car. Luckily, in modern cars, electronic stability control can change the torque on the wheels individually to prevent an accident.

Summary

In this paper, we discussed some basics of the physics of driving. We have shown how simulations can be used to demonstrate and even measure certain parameters of a starting car. We have presented how to create a simple model car in Algodoo and investigated the influence of the friction coefficient between the wheels and the ground, which axle is driven, as well as the torque and frequency of the motor. We have shown a simple derivation that the maximum possible acceleration is limited by friction if the torque on a wheel divided by the radius of the wheel is bigger than the maximum value of the friction force; otherwise, the acceleration is limited by the torque [see Eq. (10)]. The calculations were in good agreement with the results of the simulation. We found that the two kinds of motions differ visually since in the first case the car starts with slipping wheels. Algodoo offers many different possibilities for the interactive investigation of this phenomenon, some of which we have presented here. This study might serve as the basis for designing student projects or lessons on this challenging topic.

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